加速された水素様重イオンからの高エネルギー光渦の放射
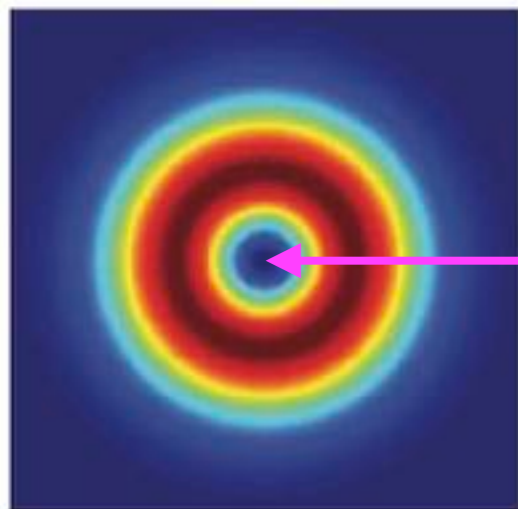
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日本物理学会 2020年秋季大会, 筑波大(オンライン), 2020/09

Twisted photons
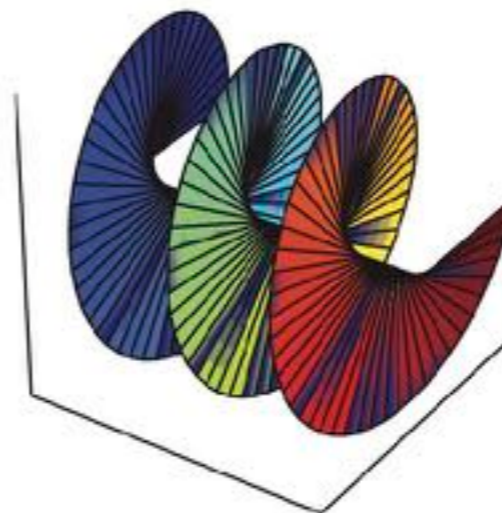
Orbital angular momentum (OAM) of light

winding field phase ~ $e^{im\varphi}$

G. Molina-Terriza et al.
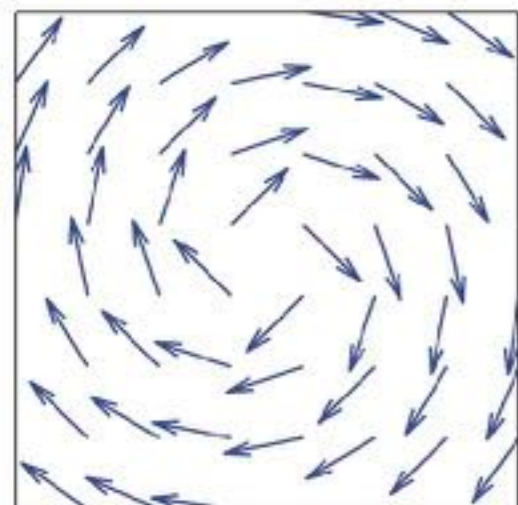Nat. Phys. 3, 305 (2007)
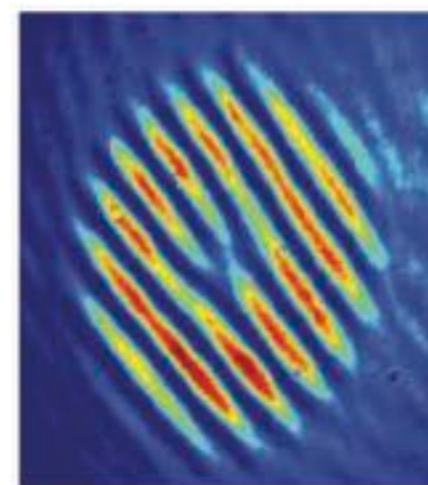

field intensity

phase
singularity

wave front

helicoid

transvers
Poynting vector

interference
pattern with PW

computer-generated hologram

Energy up-conversion with boosted ions

Rayleigh scattering:

$$\gamma_i + |g\rangle \to |e\rangle \to |g\rangle + \gamma_f$$

Boosted ions: $E = \gamma M$

e.g. $\gamma \sim 10^3$ @LHC
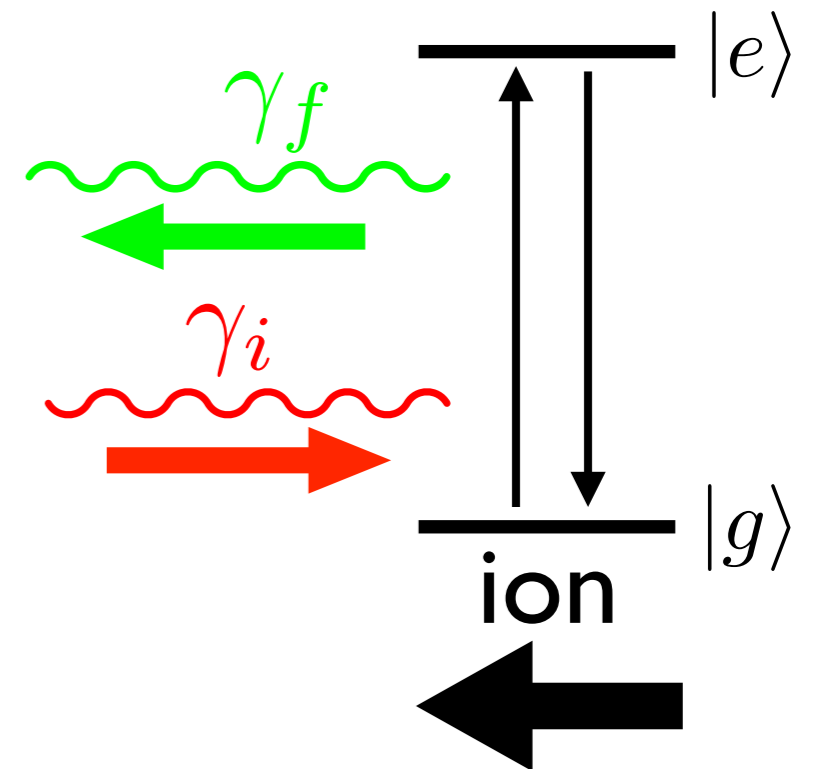
Level splitting: E_{eg}

binding energy of H-like ion $= (Z^2/n^2)13.6$ eV

Resonance condition: $2\gamma\omega_i \simeq E_{eg}$

$\omega_i \sim 1\text{-}10$ eV $\longrightarrow$ $Z^2/2\gamma \sim 0.1\text{-}1$

heavy ion

Up-conversion:

$$\omega_f^{\max} \simeq 2\gamma E_{eg} \simeq 4\gamma^2\omega_i \sim 0.1\text{-}1 \text{ GeV}(2\gamma/10^4)^2$$

Bessel beam

Twisted photon as a superposition of plane waves

PW: $A^\mu_{\boldsymbol{k}\lambda}(t,\boldsymbol{x}) = \varepsilon^\mu_\lambda(\boldsymbol{k})e^{-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}/\sqrt{2\omega}$

Twisted photon (Bessel beam):

$$A^\mu_{mk_T k_z \lambda}(t,\boldsymbol{x}) := \int a_{mk_T}(\boldsymbol{k}_T)A^\mu_{\boldsymbol{k}\lambda}(t,\boldsymbol{x})dk_T^2/(2\pi)^2$$

$$a_{mk_T}(\boldsymbol{k}_T) := (-i)^m e^{im\varphi_k}\sqrt{2\pi/k_T}\,\delta(|\boldsymbol{k}_T| - k_T)$$

$$A^\mu_{mk_T k_z \lambda}(t,\boldsymbol{x}) = -i\lambda\sqrt{k_T/4\pi\omega}\,e^{-i(\omega t - k_z z)}\left[e^{i(m-\lambda)\varphi}\cos^2\frac{\theta_k}{2}J_{m-\lambda}(k_T\rho)\eta^\mu_\lambda\right.$$

$$\left. - e^{i(m+\lambda)\varphi}\sin^2\frac{\theta_k}{2}J_{m+\lambda}(k_T\rho)\eta^\mu_{-\lambda} + \frac{i}{\sqrt{2}}e^{im\varphi}\sin\theta_k J_m(k_T\rho)\eta^\mu_0\right]$$

$$\eta^\mu_\lambda := (0,-\lambda,-i,0)/\sqrt{2}\,,\ \ \eta^\mu_0 := (0,0,0,1)$$

$$\sin\theta_k := |\boldsymbol{k}_T|/|\boldsymbol{k}| \quad \text{pitch angle}$$

Heavy ion excitation by twisted photons

Relativistic effects $\sim O(Z\alpha)$

Dirac theory of Hydrogen-like ion:

interaction hamiltonian $\quad H_I = e\,\boldsymbol{\alpha}\cdot\boldsymbol{A}$

wave function
$$\psi(\boldsymbol{x}) = \begin{pmatrix} \dfrac{G(r)}{r}\mathcal{Y}^{j_3}_{j\ell_A}(\theta,\varphi) \\[2mm] i\dfrac{F(r)}{r}\mathcal{Y}^{j_3}_{j\ell_B}(\theta,\varphi) \end{pmatrix}$$
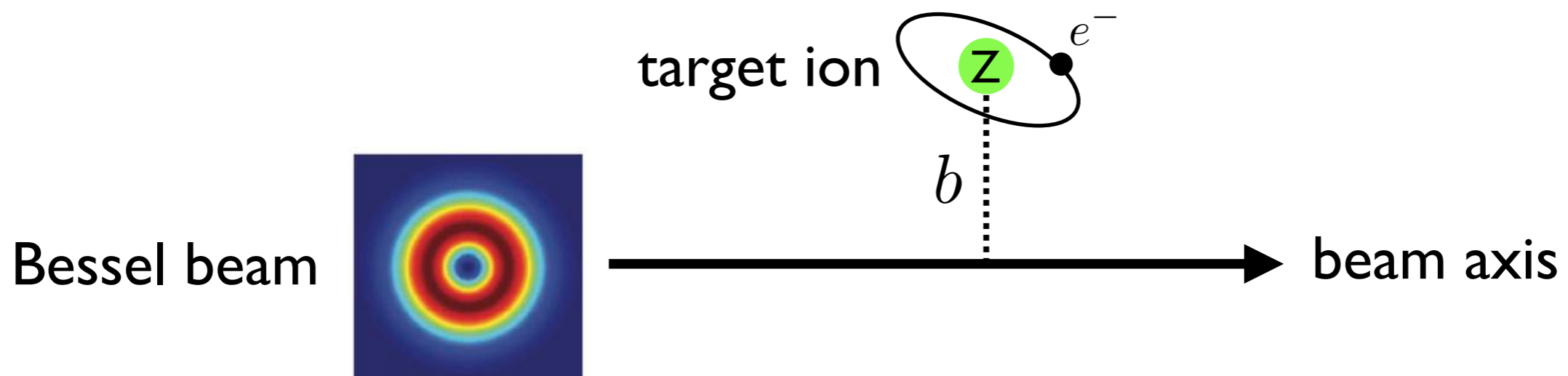
transition matrix element: $\ |i\rangle \to |f\rangle$

$$\mathcal{M}_{fi} = e\int d^3x\,\psi^\dagger_f(\boldsymbol{x})\boldsymbol{\alpha}\psi_i(\boldsymbol{x})\cdot\boldsymbol{A}(t,\boldsymbol{x})e^{i\omega t}$$

Twisted photon amplitude

a superposition of plane wave amplitudes

$$\mathcal{M}_{fi}^{(\mathrm{tw})} = (-i)^{2m+m_i-m_f} \sqrt{\frac{k_T}{2\pi}}\, e^{i(m+m_i-m_f)\phi_b} J_{m+m_f-m_i}(k_T b)$$

$$\times \sum_{m_f',m_i'} d_{m_f m_f'}^{j_f}(\theta_k) d_{m_i m_i'}^{j_i}(\theta_k) \mathcal{M}_{m_f' m_i'}^{(\mathrm{pl})}$$
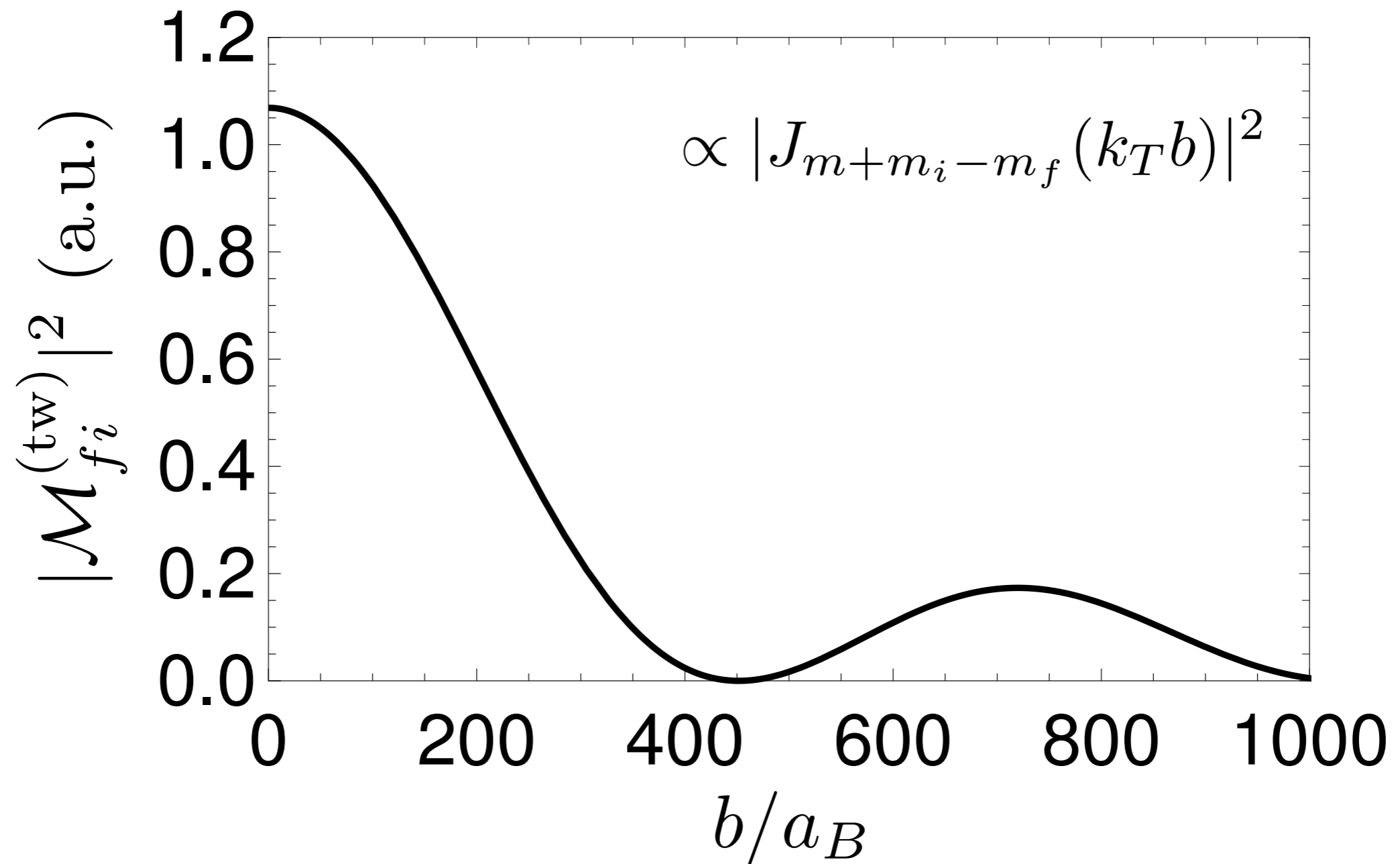
impact parameter: $\boldsymbol{b} = b(\cos\phi_b, \sin\phi_b, 0)$

target ion $\quad$ Z $\quad e^-$

b

Bessel beam $\qquad\qquad$ beam axis

Dependence on b: H-like Pb (Z=82)

$$1\mathrm{s}_{1/2}(m_i = 1/2) \to 3\mathrm{d}_{5/2}(m_f = 5/2)\,,\ \gamma = 4600$$

$$\theta_k = 1/\gamma\,,\ \lambda = +1\,,\ m = 2$$

$$\propto |J_{m+m_i-m_f}(k_T b)|^2$$

y-axis: $|\mathcal{M}_{fi}^{(\mathrm{tw})}|^2$ (a.u.)

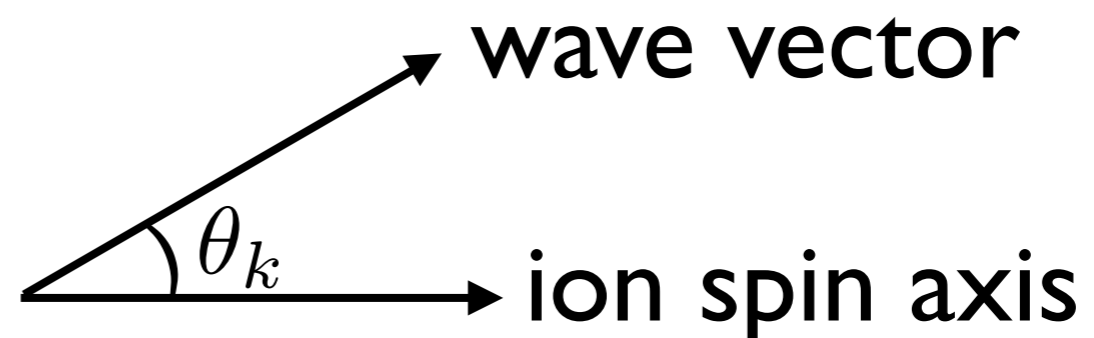x-axis: b/a_B

Cross section

Randomly distributed ions

⟹ average over the impact parameter $(k_T R \gg 1)$

$$\sigma = \frac{4}{(\Gamma_f + \Gamma_L)\cos\theta_k} |\mathcal{M}_{fi}^{(\mathrm{pl})}(\theta_k)|^2 \quad \textbf{on-resonance}$$

$\Gamma_f :$ natural width, $\Gamma_L :$ laser width

$$\mathcal{M}_{fi}^{(\mathrm{pl})}(\theta_k) = \sum_{m'_f, m'_i} d_{m_f m'_f}^{j_f}(\theta_k) d_{m_i m'_i}^{j_i}(\theta_k) \mathcal{M}_{m'_f m'_i}^{(\mathrm{pl})}$$
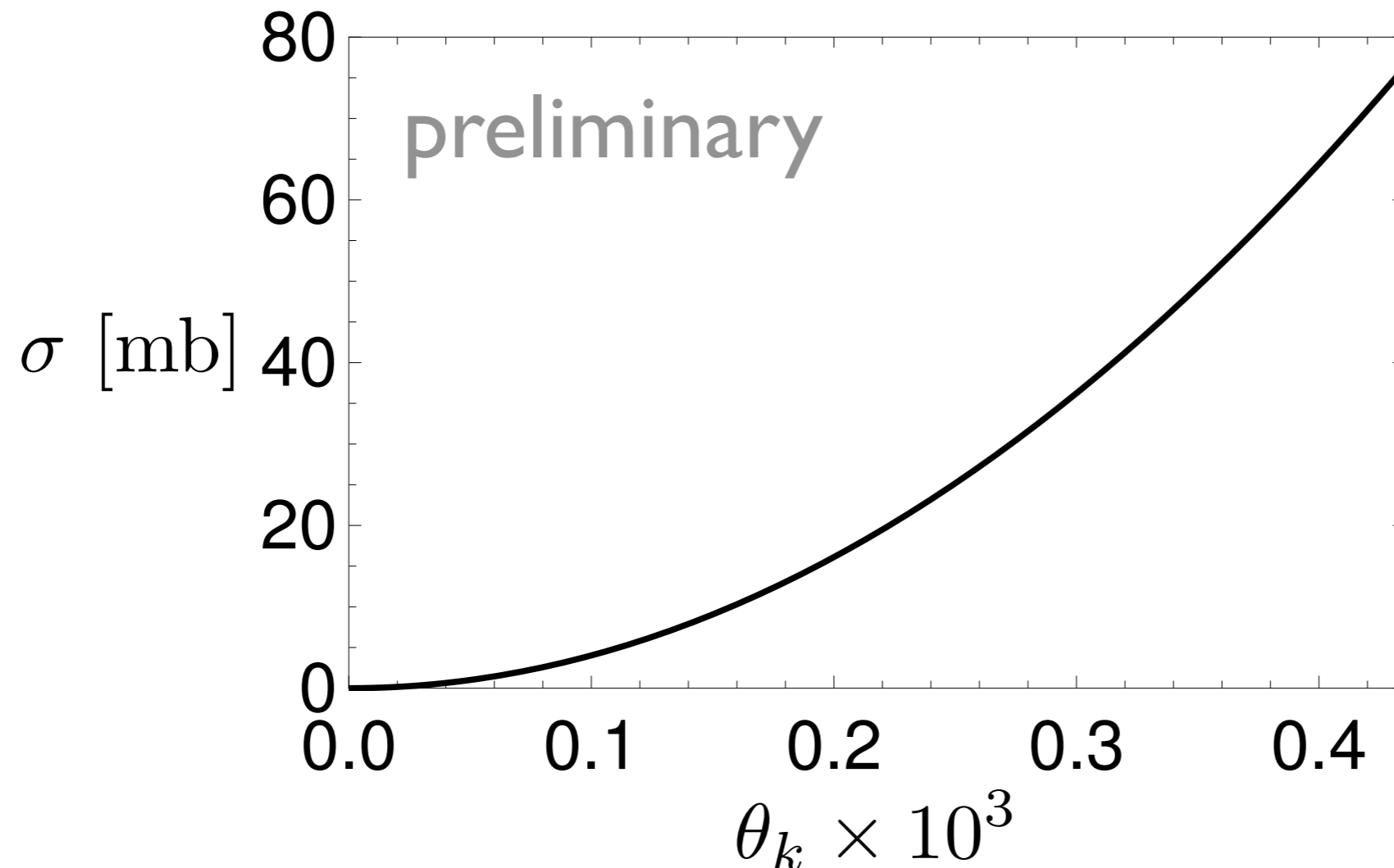
oblique plane wave amplitude

wave vector

θ_k

ion spin axis

H-like Pb (Z=82)

$$1\mathrm{s}_{1/2}(m_i = 1/2) \to 3\mathrm{d}_{5/2}(m_f = 5/2)\,,\ \gamma = 4600$$

preliminary

(figure: plot of σ [mb] vs $\theta_k \times 10^3$)

De-excitation to emit energetic twisted photons

$$\mathrm{BR}(3\mathrm{d}_{5/2} \to 1\mathrm{s}_{1/2}) \simeq 0.045$$

Summary

✶ Twisted photons: OAM of light

✶ Energy up-conversion with boosted ions

$$\omega_f^{\max} \simeq 4\gamma^2 \omega_i \, , \ 2\gamma \sim 10^4 \quad \text{optical} \rightarrow \text{gamma ray}$$

✶ Twisted photon process: forbidden → allowed

✶ Absorption & emission rates
Relativistic calculation for heavy ions
H-like Pb, typical CS x BR ~ 1 mb

✶ Twisted photon flux
laser and ion beam spec. to be studied