Physics
OSAKA UNIVERSITY

フレーバー物理の基礎

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1. Introduction

歴史と背景

θ - τ 問題: $\theta^{\pm}, \ \tau^{\pm} \quad m_\theta = m_\tau \simeq 1000 m_e \, , \ \Gamma_\theta = \Gamma_\tau$

$\theta \to 2\pi \, , \tau \to 3\pi$

1956: T.D. Lee, C.N. Yang (ノーベル賞1957)

弱い相互作用におけるパリティーの破れ

1957: C.S. Wu et al.

^{60}Co β 崩壊実験 パリティーの破れの発見

1957: L. D. Landau

$K_S \to \pi\pi, \ K_L \to \pi\pi\pi \qquad \tau_{K_S} \ll \tau_{K_L}$

中性K中間子崩壊におけるCP保存

1961: S. L. Glashow (ノーベル賞1979)

$SU(2) \times U(1)$

1964: J. W. Cronin, V. L. Fitch, … (ノーベル賞1980)

中性K中間子崩壊におけるCPの破れ

$$K_L \to \pi^+\pi^- \quad \mathrm{BR} \simeq 2\times 10^{-3}$$

1964: F. Englert and R.Brout, P. Higgs (ノーベル賞2013)

ゲージ対称性の自発的破れとゲージボソン質量

Higgs粒子

1967: S. Weinberg (ノーベル賞1979)

1968: A. Salam (ノーベル賞1979)

electroweak theory

1970: S.L.Glashow, J. Iliopoulos, L. Maiani

FCNCに基づくチャームクォークの予言

1971: G. ’t Hooft (ノーベル賞1999)
ゲージ理論の繰り込み[5]

1973: M. Kobayashi, T. Maskawa (ノーベル賞2008)
CPを破る6クォーク模型

1974: S.Ting, B. Richter (ノーベル賞1976)
J/Ψ (チャーム)の発見

1975: M. L. Perl (ノーベル賞1995)　タウの発見

1977: L. M. Lederman　ボトムの発見

1983: UA1, UA2　W, Zの発見
(C. Rubia, S. van der Meer,ノーベル賞1984)

1987: ARGUS　B^0–$\bar{B}^0$ 混合の発見

1989: CLEO　$b \to u$ 遷移の発見

1994: CDF, D0　トップの発見

2002: Belle, BABAR

B中間子崩壊におけるCPの破れの確立

2012: ATLAS, CMS　ヒッグスボソンの発見

標準模型の確立

The standard model of particle physics

Gauge group: $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$

Particle contents

	l_L	e_R	q_L	u_R	d_R	G	W	B	Φ
$SU(3)_C$	1	1	3	3	3	8	1	1	0
$SU(2)_L$	2	1	2	1	1	1	3	1	2
$U(1)_Y$	$-1/2$	-1	$1/6$	$2/3$	$-1/3$	0	0	0	$1/2$

fermions gauge bosons Higgs

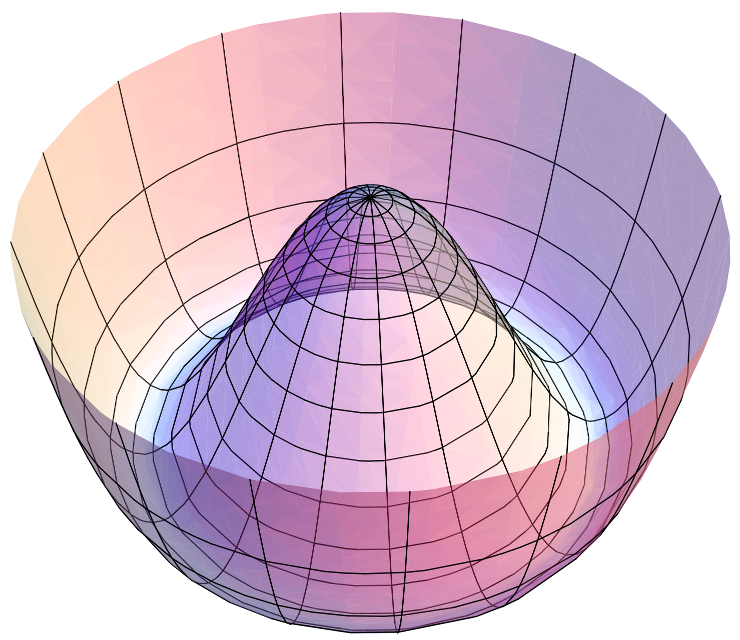
$$l_L := \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad q_L := \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$Q = T_L^3 + Y \ (Y = \langle Q \rangle)$$

three generations of fermions

Spontaneous symmetry breaking

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow[\langle\Phi\rangle]{} SU(3)_C \times U(1)_{em}$$



$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = (\sqrt{2}G_F)^{1/2} = 246\ \mathrm{GeV}$$

Plan

1. Introduction (5)
2. Quark mass, mixing and CP violation (10)
3. Meson-antimeson mixing (9)
4. CP violation in B decays (4)
5. Summary (1)

2. Quark mass, mixing and CP violation

Yukawa interaction

$$-\mathcal{L}_Y = \bar{l}_{Li}\, y^e_{ij}\, e_{Rj}\, \Phi + \bar{q}_{Li}\, y^d_{ij}\, d_{Rj}\, \Phi + \bar{q}_{Li}\, y^u_{ij}\, u_{Rj}\, \tilde{\Phi} + \text{h.c.}$$

y^f $(f = e, d, u)$: complex 3×3 matrix

$$\Phi := \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \tilde{\Phi} := i\tau_2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \quad (i\tau_2 = \epsilon)$$

CP transformation

$$y_{ijk}\bar{\psi}_i\psi_j\phi_k + y^*_{ijk}\bar{\psi}_j\psi_i\phi^*_k \xrightarrow{CP} y_{ijk}\bar{\psi}_j\psi_i\phi^*_k + y^*_{ijk}\bar{\psi}_i\psi_j\phi_k$$

h.c.

Potentially CP violating, if y's have complex phases.

Rephasing of ψ's and ϕ's may remove all complex phases in y's.

Quark mass and mixing

$$-\mathcal{L}_Y = \bar{l}'_{Li}\, y^e_{ij}\, e'_{Rj}\, \Phi + \bar{q}'_{Li}\, y^d_{ij}\, d'_{Rj}\, \Phi + \bar{q}'_{Li}\, y^u_{ij}\, u'_{Rj}\, \tilde{\Phi} + \text{h.c.} \qquad f' : \text{gauge (weak) basis}$$

Electroweak symmetry breaking $\langle \Phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

$$\Rightarrow \quad -\mathcal{L}_{FM} = \bar{e}'_L\, M^e\, e'_R + \bar{d}'_L\, M^d\, d'_R + \bar{u}'_L\, M^u\, u'_R + \text{h.c.}$$

fermion mass matrices: $M^f_{ij} := \dfrac{v}{\sqrt{2}}\, y^f_{ij} \quad (f = e, d, u)$

Mass basis $f'_{L,R} = U^f_{L,R} f_{L,R}\,,\ U^f_{L,R} :\ 3 \times 3$ unitary (mixing) matrix

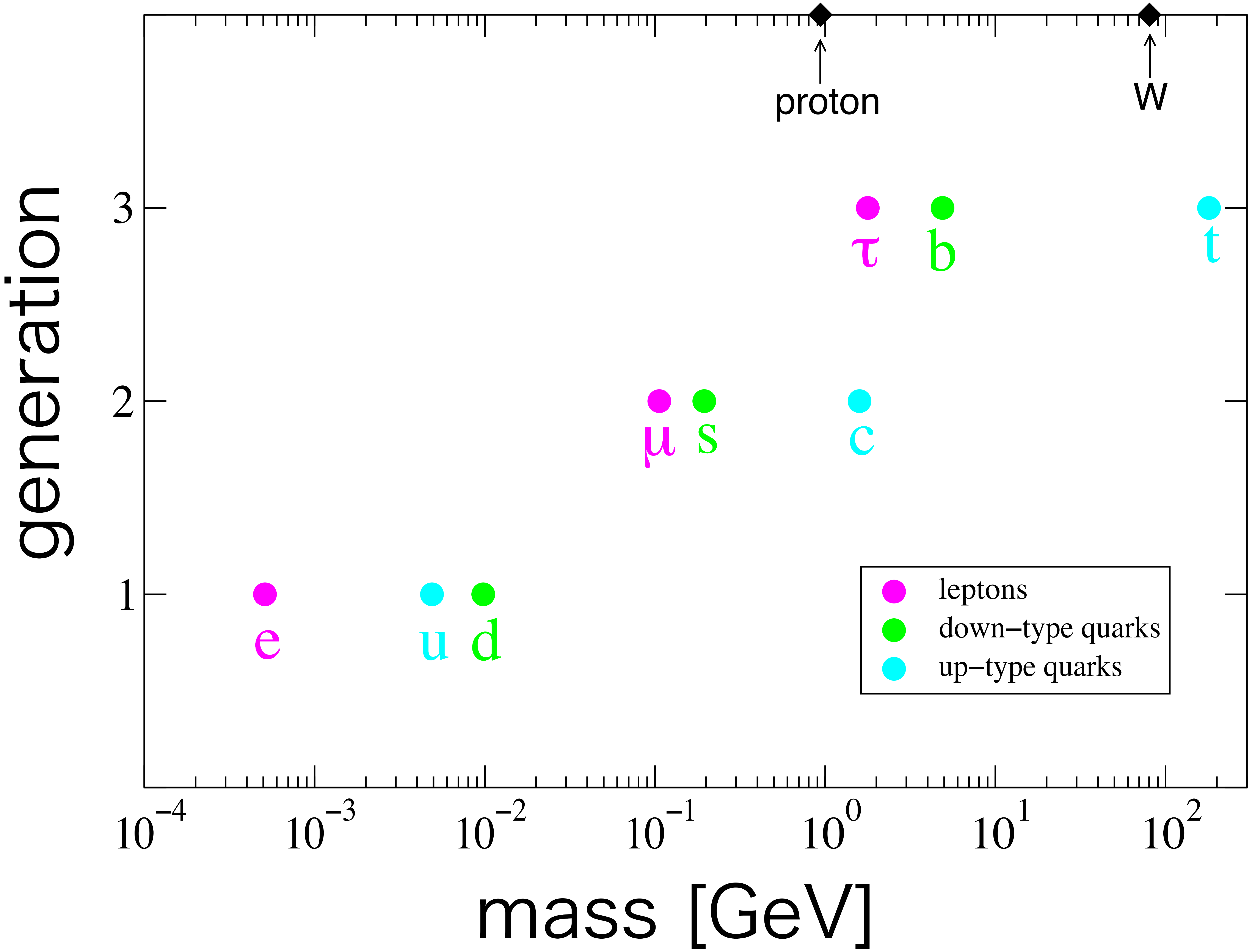
$$U^{e\dagger}_L M^e U^e_R = D^e = \text{diag.}(m_e, m_\mu, m_\tau)$$

$$U^{d\dagger}_L M^d U^d_R = D^d = \text{diag.}(m_d, m_s, m_t)$$

$$U^{u\dagger}_L M^u U^u_R = D^u = \text{diag.}(m_u, m_c, m_t)$$

$$\Rightarrow \quad -\mathcal{L}_{FM} = \bar{e}_L\, D^e\, e_R + \bar{d}_L\, D^d\, d_R + \bar{u}_L\, D^u\, u_R + \text{h.c.}$$

Fermion mass hierarchy



No explanation in the SM

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Cabibbo-Kobayashi-Maskawa (CKM) mixing

$$q'_L = \begin{pmatrix} u'_L \\ d'_L \end{pmatrix} = \begin{pmatrix} U^u_L u_L \\ U^d_L d_L \end{pmatrix} \qquad U^u_L \neq U^d_L \Longrightarrow \text{noncommuting with } \tau_{1,2} \text{ of } SU(2)_L$$

Charged current interaction

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} W^+_\mu \left(\bar{u}'_L \gamma^\mu d'_L + \bar{\nu}'_L \gamma^\mu e'_L\right) + \text{h.c.}$$

gauge (weak) basis

$$= \frac{g}{\sqrt{2}} W^+_\mu \left(\bar{u}_L \gamma^\mu U^{u\dagger}_L U^d_L d_L + \bar{\nu}_L \gamma^\mu e_L\right) + \text{h.c.}$$

mass basis

$V_{\rm CKM} := U^{u\dagger}_L U^d_L$ Cabibbo-Kobayashi-Maskawa (CKM) matrix

$N_G \times N_G$ unitary matrix $\qquad$ N_G^2 real parameters

rephasing of the quark fields	$2N_G \underbrace{-1}_{\text{overall}}$	unphysical
$O(N)$ rotation (mixing angles)	$N_G(N_G-1)/2$	physical
CP violating complex phases	$(N_G-1)(N_G-2)/2$	physical

CP violation $\Longrightarrow N_G \geq 3$

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PDG parametrization

$$V_{\rm CKM} =: \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

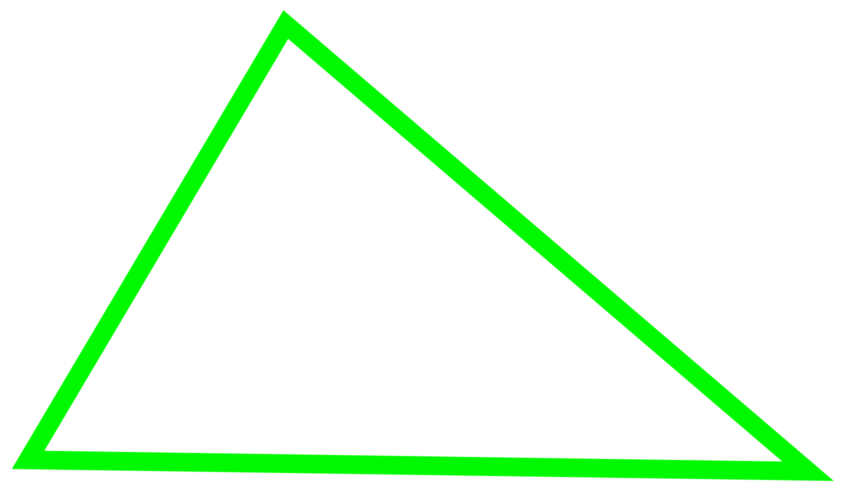
$$=: \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} = \cos\theta_{ij}\,,\ \ s_{ij} = \sin\theta_{ij}\,,\ \ 0 < \theta_{ij} < \pi/2\,,\ \ 0 < \delta < 2\pi$$

Unitarity triangle

$i \neq j$ のとき, $\displaystyle\sum_{k=1}^{3} V_{ki}^* V_{kj} = 0$ 複素平面上の三角形

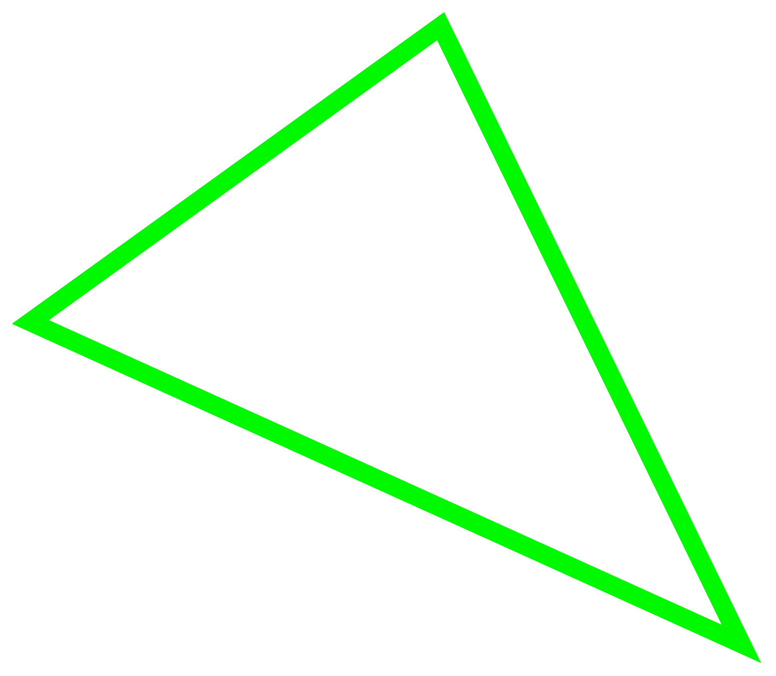


ユニタリティートライアングル

三角形の形や面積は, クォークの位相に依らない.

位相の再定義は, 三角形全体の回転に対応.

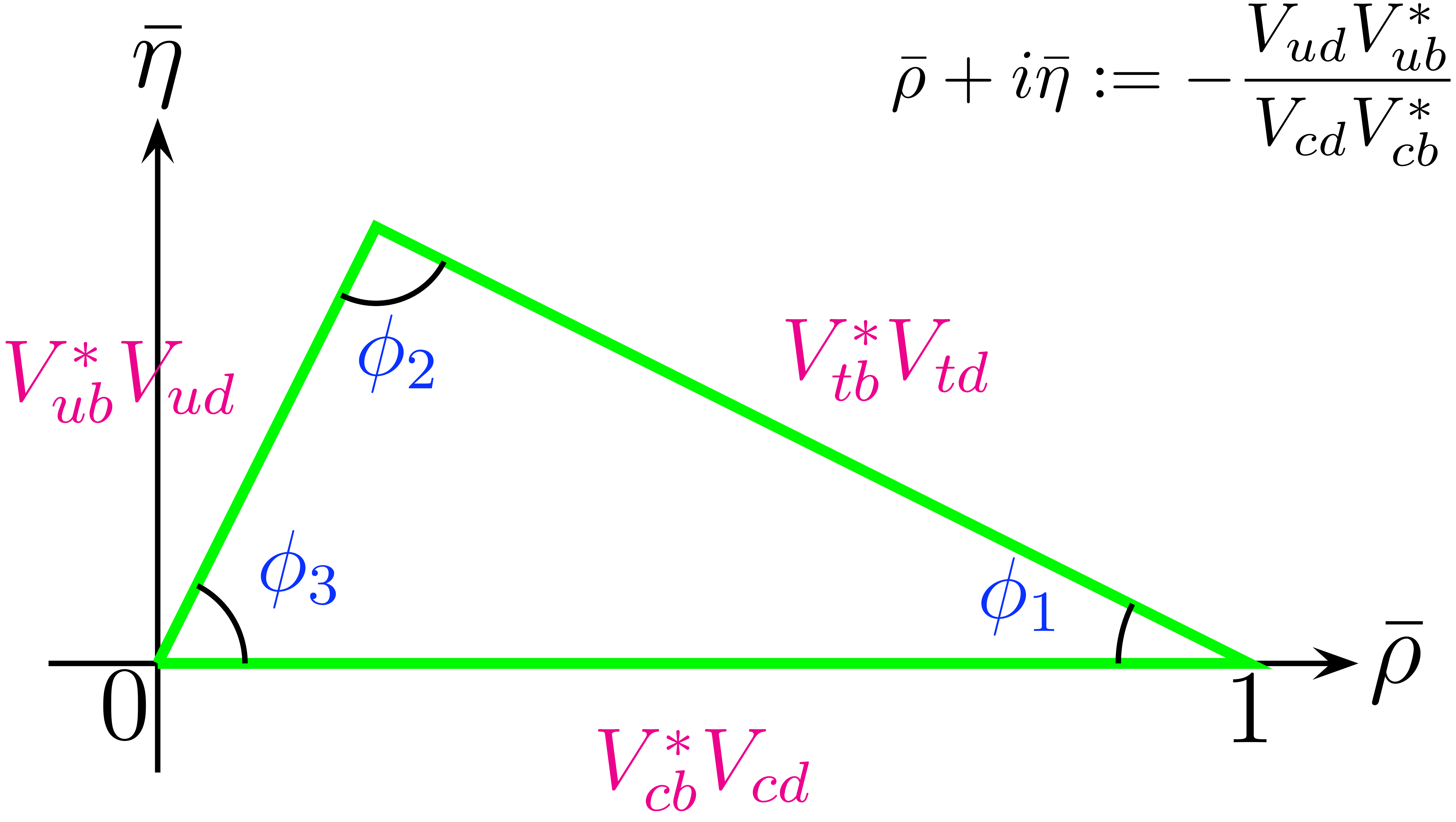
$$\sum_{k=1}^{3} e^{i\varphi_k} V_{ki}^* e^{-i\varphi_i} e^{-i\varphi_k} V_{kj} e^{i\varphi_j} = e^{i(\varphi_j - \varphi_i)} \sum_{k=1}^{3} V_{ki}^* V_{kj} = 0$$



B meson and unitarity triangle

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

$$\bar{\rho} + i\bar{\eta} := -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$



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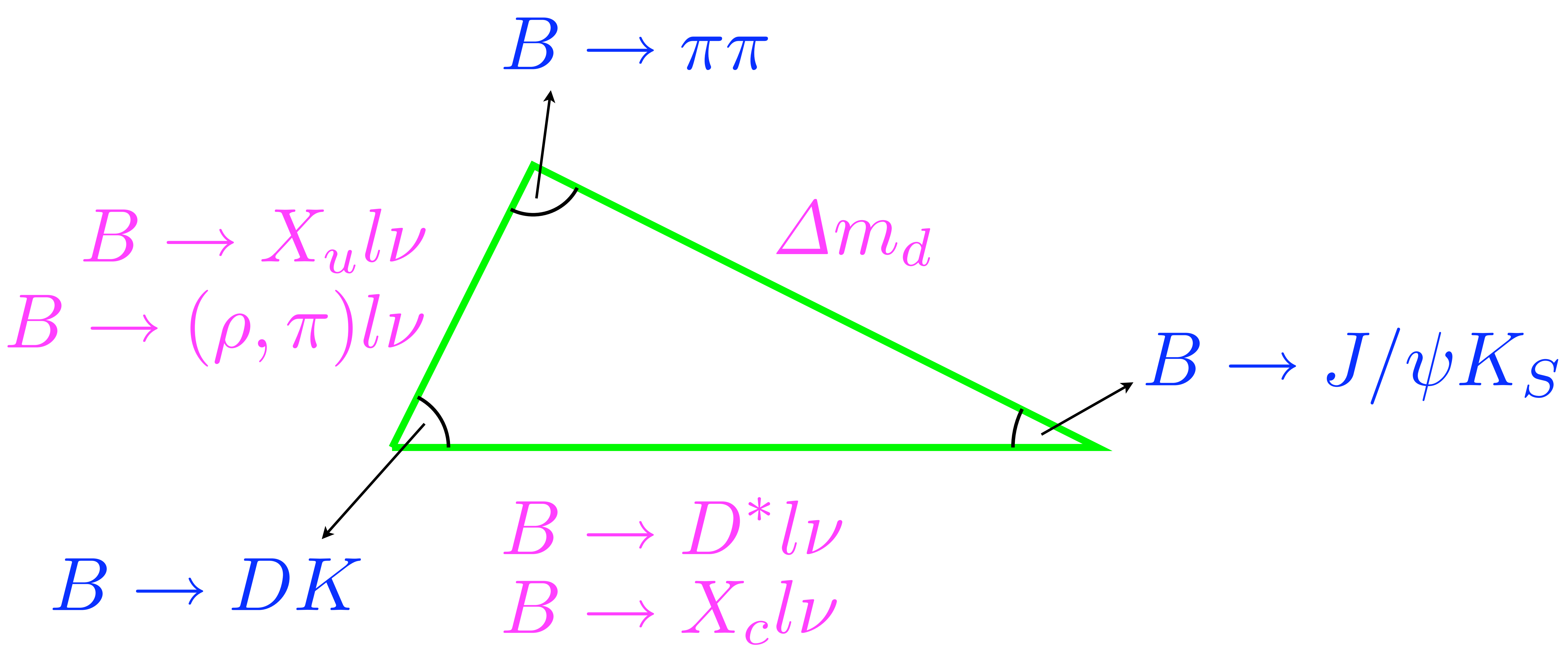
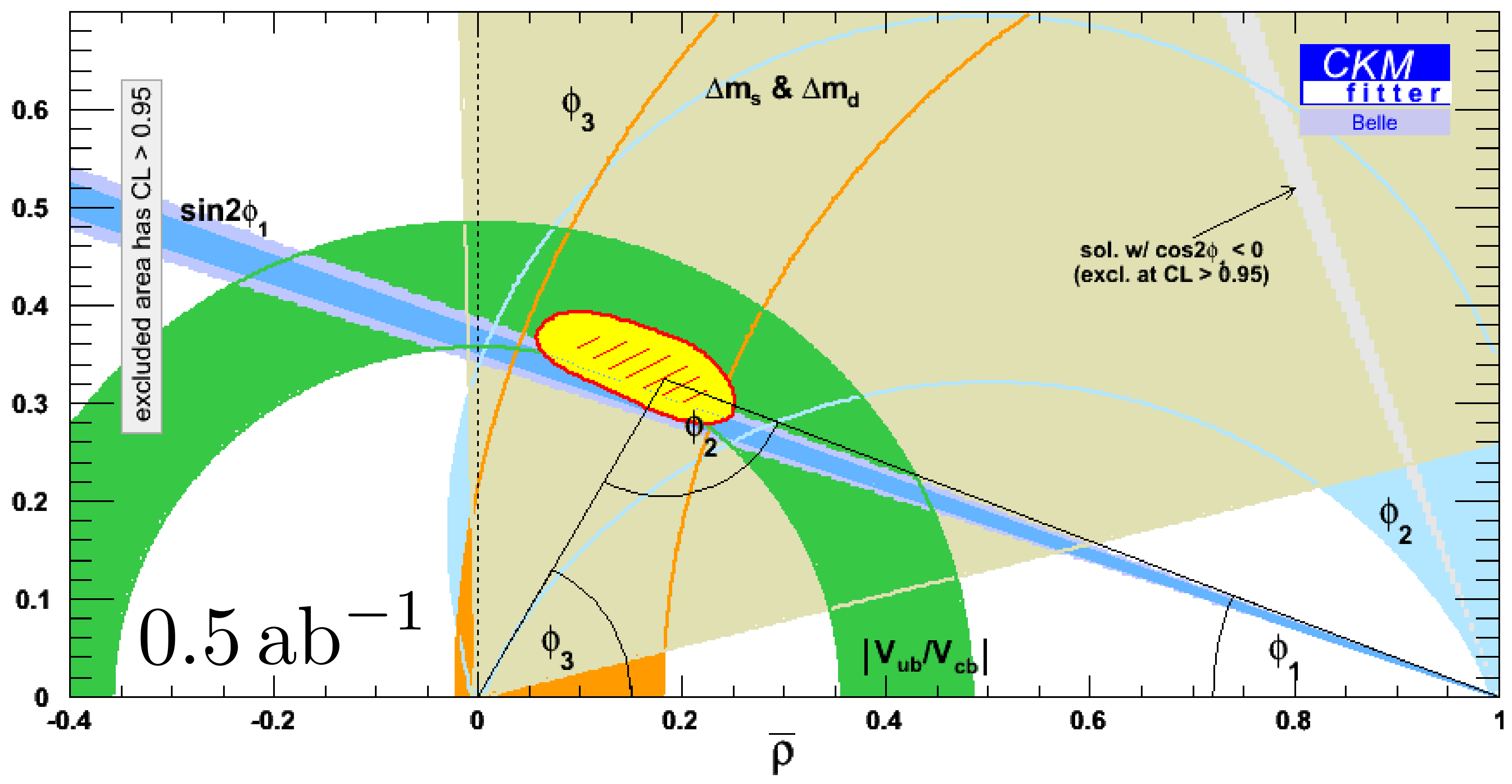


Is the unitarity triangle closed?

Bファクトリー

~10%



スーパーBファクトリー

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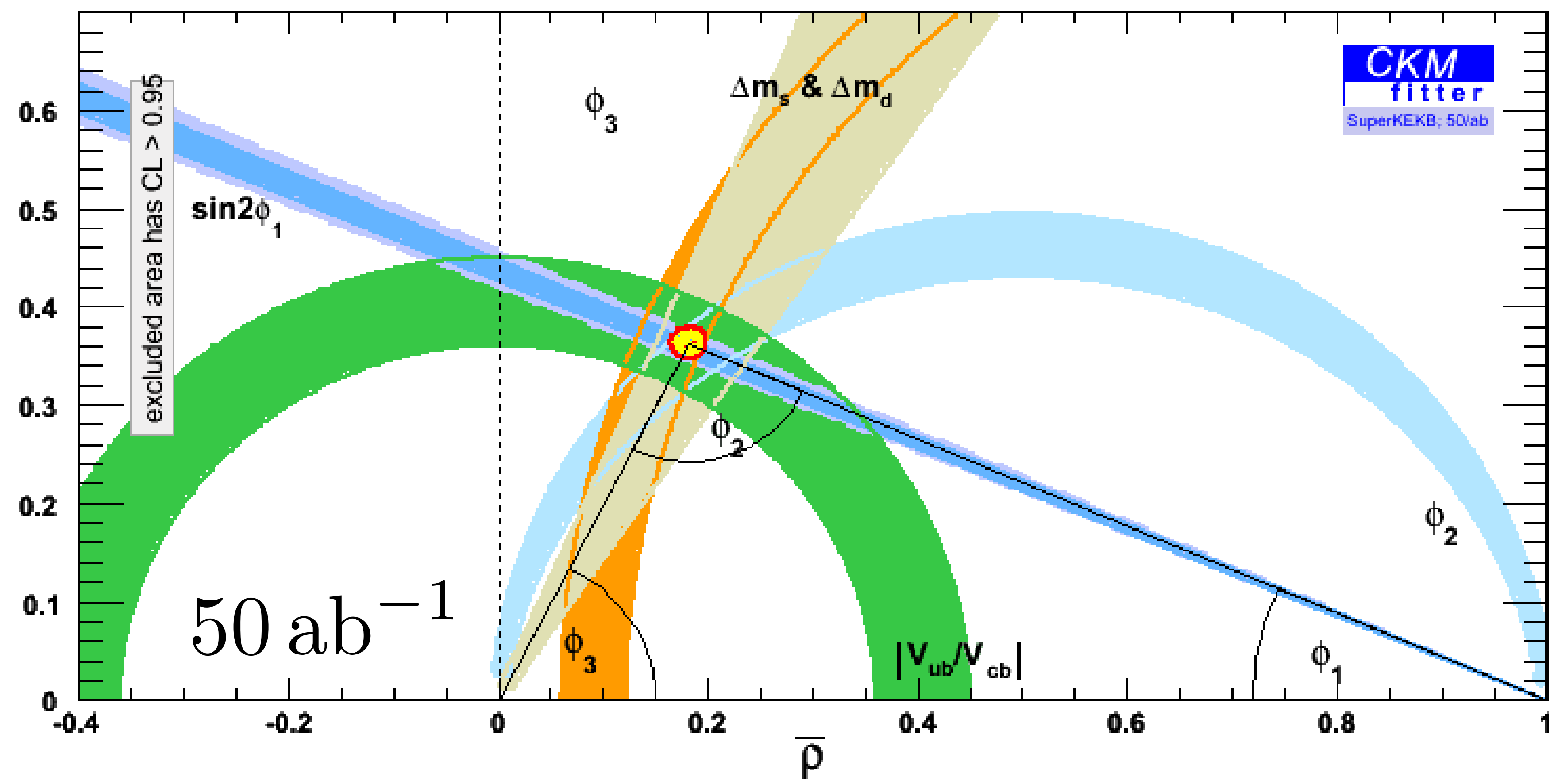

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Flavor changing neutral current (FCNC)

$$q'_L = \begin{pmatrix} u'_L \\ d'_L \end{pmatrix} = \begin{pmatrix} U^u_L u_L \\ U^d_L d_L \end{pmatrix} \qquad U^u_L \neq U^d_L \Longrightarrow \text{noncommuting with } \tau_{1,2} \text{ of } SU(2)_L$$

$$\text{but, commuting with } \tau_3$$

$$-\mathcal{L}_{\rm NC} = g_z Z_\mu \sum_f \bar{f}\gamma^\mu (T^3_L - Q\sin^2\theta_w) f$$

No tree-level FCNC in the SM

→ FCNC's are loop-induced and suppressed.

good for new physics search

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3. Meson-antimeson mixing

P^0–$\bar{P}^0$ mixing

$P^0 = K^0(\bar{s}d), D^0(\bar{u}c), B^0(\bar{b}d), B_s^0(\bar{b}s)$

Two-level quantum system, but they decay.

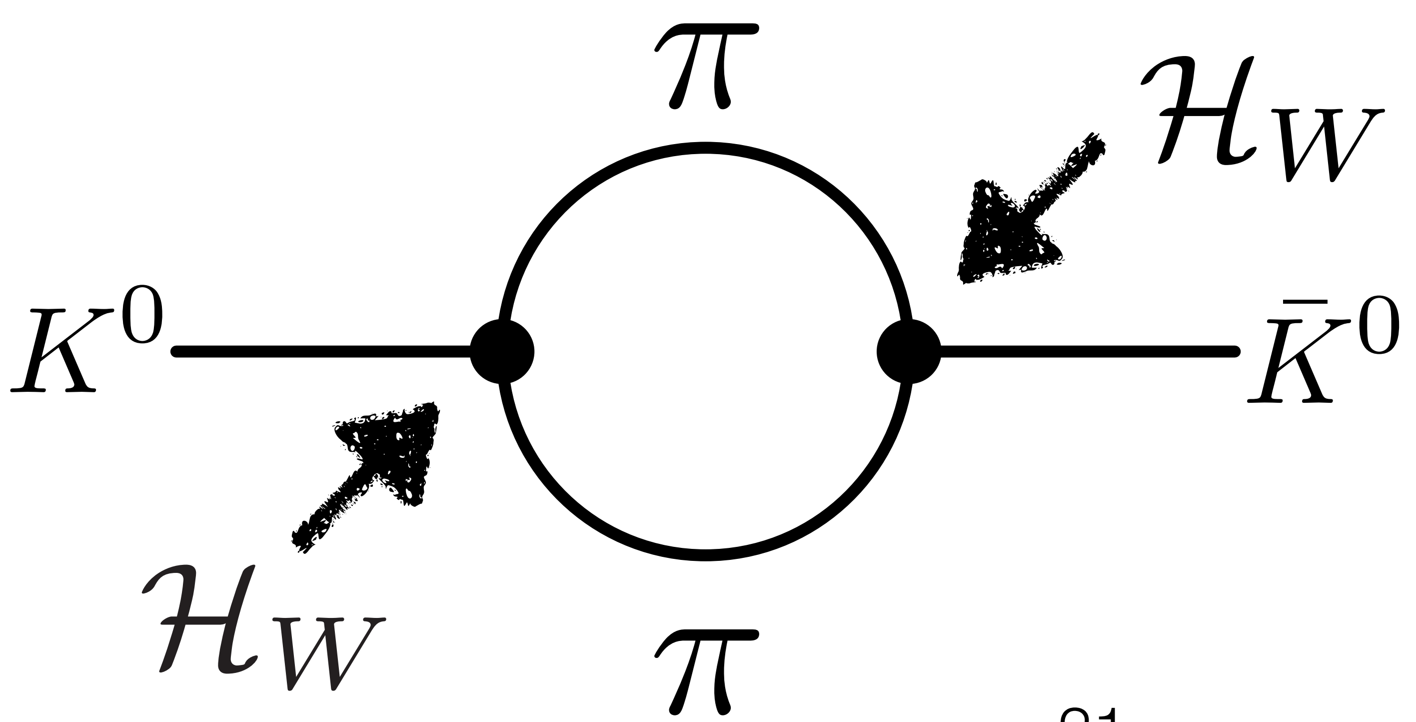
→ open quantum system   cf. qubit

Charged current

changing flavor by one unit $\Delta s, \Delta c, \Delta b = \pm 1$

→ P^0–$\bar{P}^0$ induced by the 2nd order perturbation

e.g.



$\mathcal{H}_W \ni a_\pi^\dagger a_\pi^\dagger a_{K^0}, a_\pi^\dagger a_\pi^\dagger a_{\bar{K}^0}, \text{h.c.}$

Equation of motion

State: $|\psi(t)\rangle = \psi_P(t)|P^0\rangle + \psi_{\bar{P}}(t)|\bar{P}^0\rangle$ $\qquad |\psi_P(t)|^2 + |\psi_{\bar{P}}(t)|^2 = 1$

Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix}\psi_P(t)\\ \psi_{\bar{P}}(t)\end{pmatrix} = \begin{pmatrix}M_{11}-\frac{i}{2}\Gamma_{11} & M_{12}-\frac{i}{2}\Gamma_{12}\\ M_{21}-\frac{i}{2}\Gamma_{21} & M_{22}-\frac{i}{2}\Gamma_{22}\end{pmatrix}\begin{pmatrix}\psi_P(t)\\ \psi_{\bar{P}}(t)\end{pmatrix}$$

$M\,,\Gamma : 2\times 2$ hermite matrices

CPT invariance: $M_{11} = M_{22}(=: M_0)\,,\ \Gamma_{11} = \Gamma_{22}(=: \Gamma_0)$

$$M_{12} = \sum_n \mathcal{P}\frac{1}{m_P - E_n}\langle P^0|\mathcal{H}_W|n\rangle\langle n|\mathcal{H}_W|\bar{P}^0\rangle$$

$$\Gamma_{12} = \sum_n 2\pi\delta(m_P - E_n)\langle P^0|\mathcal{H}_W|n\rangle\langle n|\mathcal{H}_W|\bar{P}^0\rangle$$

$$\frac{1}{x \pm i\varepsilon} = \mathcal{P}\frac{1}{x} \mp i\pi\delta(x)$$

CP transformation: $CP|P^0\rangle = -|\bar{P}^0\rangle$ (in our phase convention)

CP invariance: $M_{12} = M_{21} = M_{12}^*(= \text{real})\,, \Gamma_{12} = \Gamma_{21} = \Gamma_{12}^*(= \text{real})$

Note: $$[M, \Gamma] = M_{12}\Gamma_{12} \begin{pmatrix} \frac{\Gamma_{12}^*}{\Gamma_{12}} - \frac{M_{12}^*}{M_{12}} & 0 \\ 0 & \frac{M_{12}^*}{M_{12}} - \frac{\Gamma_{12}*}{\Gamma_{12}} \end{pmatrix}$$

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Physical states and temporal evolution

Eigenvectors of $M - \frac{i}{2}\Gamma$ (Heavy, Light)

$$|P_{H,L}\rangle = p|P^0\rangle \pm q|\bar{P}^0\rangle \qquad |p|^2 + |q|^2 = 1 \qquad \frac{q}{p} = \frac{\sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}}{M_{12} - \frac{i}{2}\Gamma_{12}}$$

Eigenvalues (mass and width)

$$\lambda_{H,L} = m_{H,L} - \frac{i}{2}\Gamma_{H,L} = M_0 - \frac{i}{2}\Gamma_0 \pm \sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}$$

$$\Delta m := m_H - m_L = 2\mathrm{Re}\sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}\ (> 0)$$

$$\Delta\Gamma := \Gamma_H - \Gamma_L = -4\mathrm{Im}\sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}$$

Note: $\langle P_L|P_H\rangle = |p|^2 - |q|^2 \neq 0$

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CP limit: $|q/p| = 1\,,\ \Delta m = 2|M_{12}|\,,\ \Delta\Gamma = \Gamma_{12}\,,\ \langle P_L|P_H\rangle = 0$

$|P_\pm\rangle = (|P^0\rangle \mp |\bar{P}^0\rangle)/\sqrt{2}$ CP eigenstates

Temporal evolution

P^0 at $t=0$ $\quad |P^0(t)\rangle = g_+(t)|P^0\rangle + \frac{q}{p}g_-(t)|\bar{P}^0\rangle$

$\bar{P}^0$ at $t=0$ $\quad |\bar{P}^0(t)\rangle = \frac{p}{q}g_-(t)|P^0\rangle + g_+(t)|\bar{P}^0\rangle$

$$g_\pm(t) := \frac{1}{2}[\exp\left(-i\lambda_H t\right) \pm \exp\left(-i\lambda_L t\right)]$$

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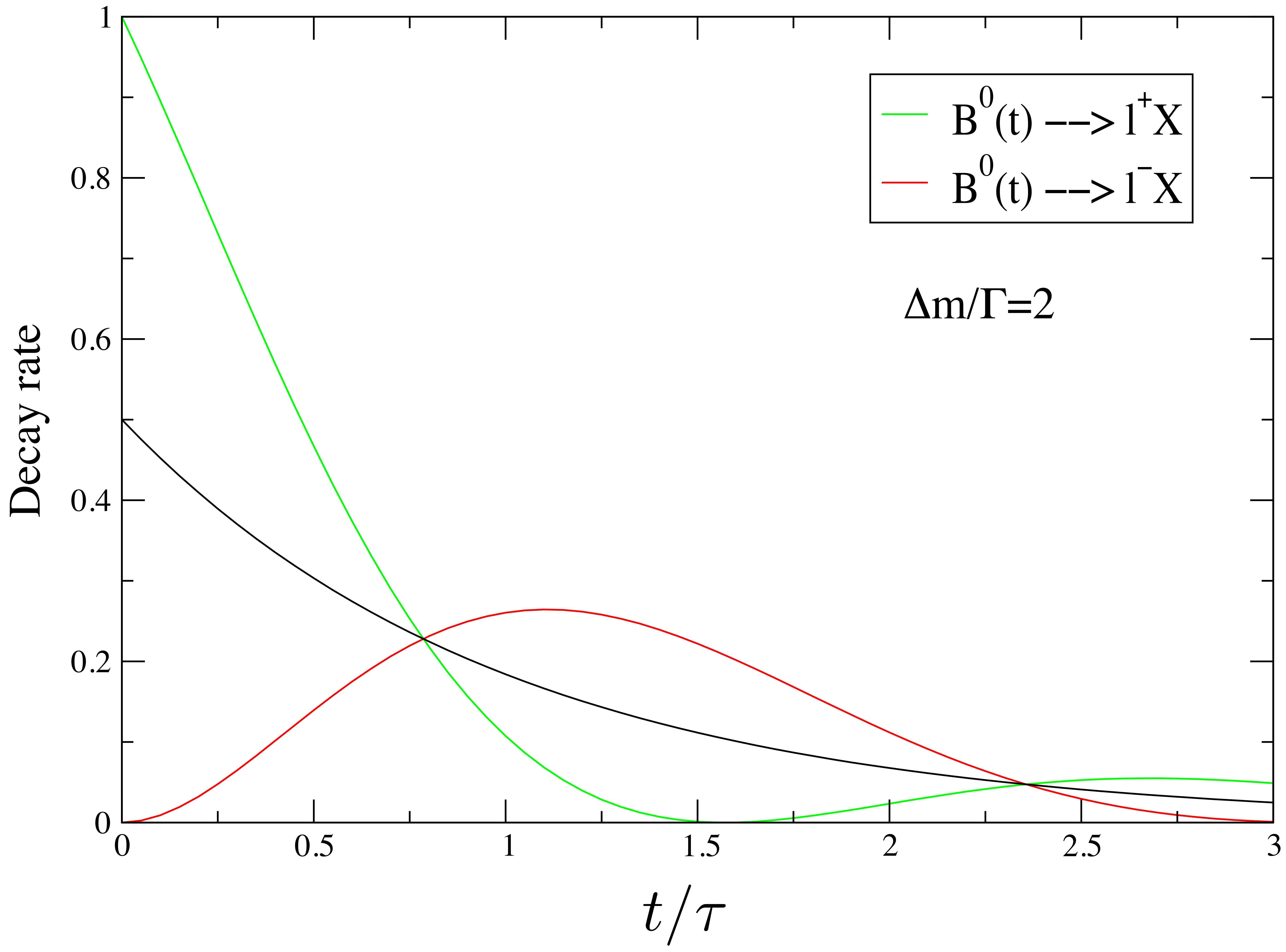
An illustration



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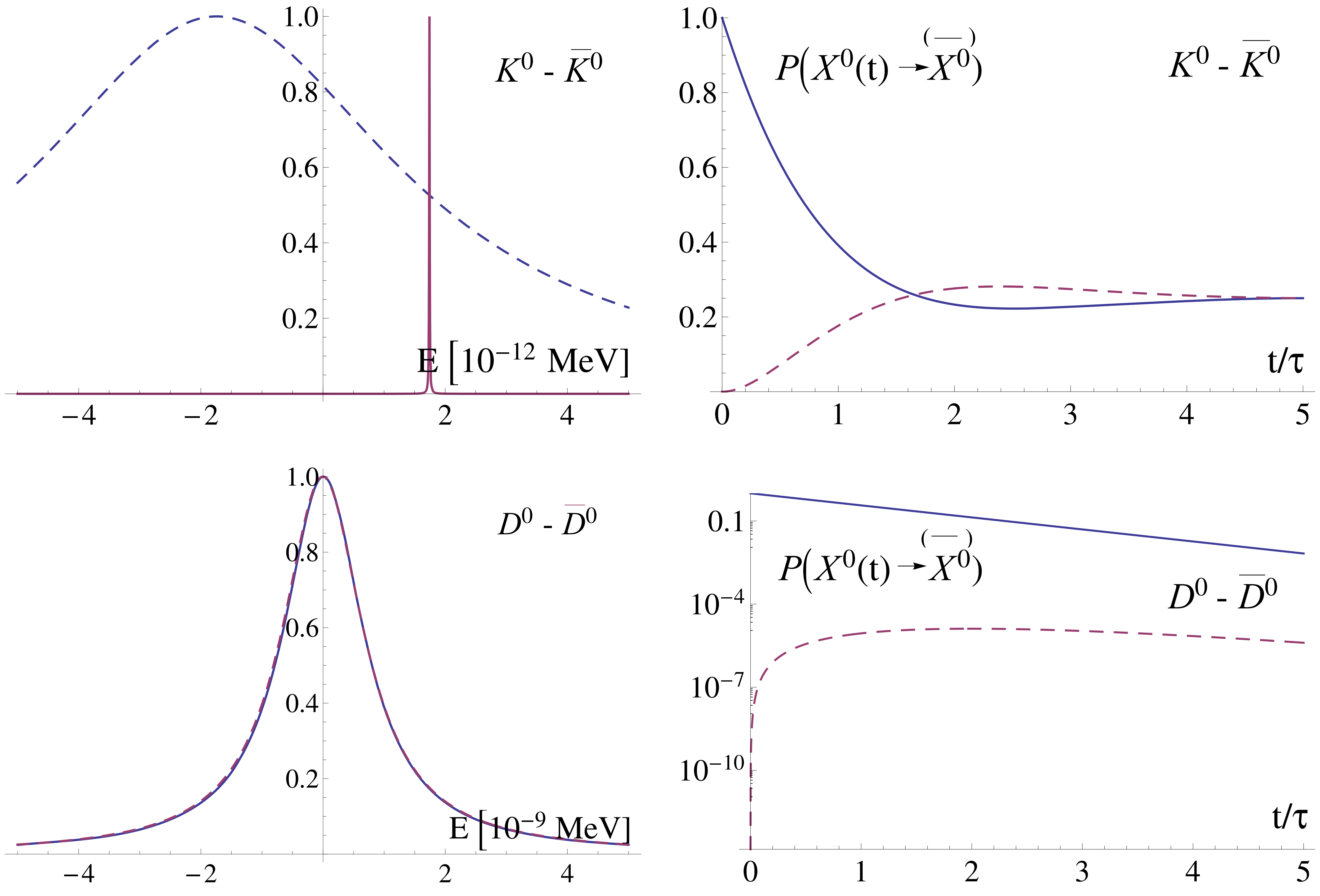
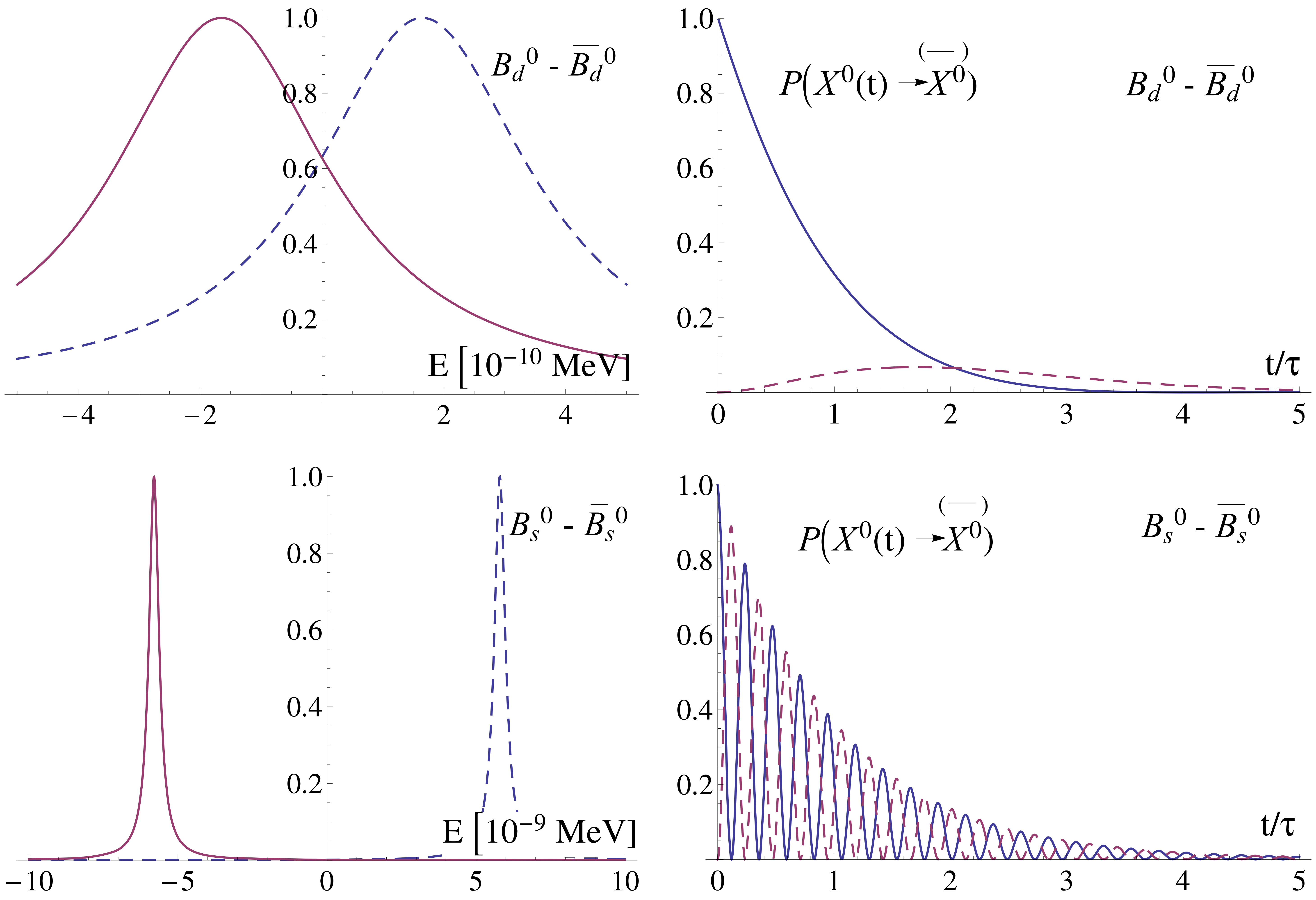
Real meson-antimeson systems



mass & width oscillation

A.J. Bevan et al. "The Physics of the B factories", EPJC74(2014)3026.



mass & width

oscillation

A.J. Bevan et al. "The Physics of the B factories", EPJC74(2014)3026.

Meson	M/MeV	Δm/MeV	Γ/MeV	$\Delta\Gamma$/MeV
K^0	497.6	3.48×10^{-12}	3.68×10^{-12}	7.34×10^{-12}
D^0	1864.9	9.45×10^{-12}	1.6×10^{-9}	2.57×10^{-11}
B_d	5279.6	3.34×10^{-10}	4.43×10^{-10}	~ 0
B_s	5366.8	1.16×10^{-8}	4.39×10^{-10}	6.58×10^{-11}

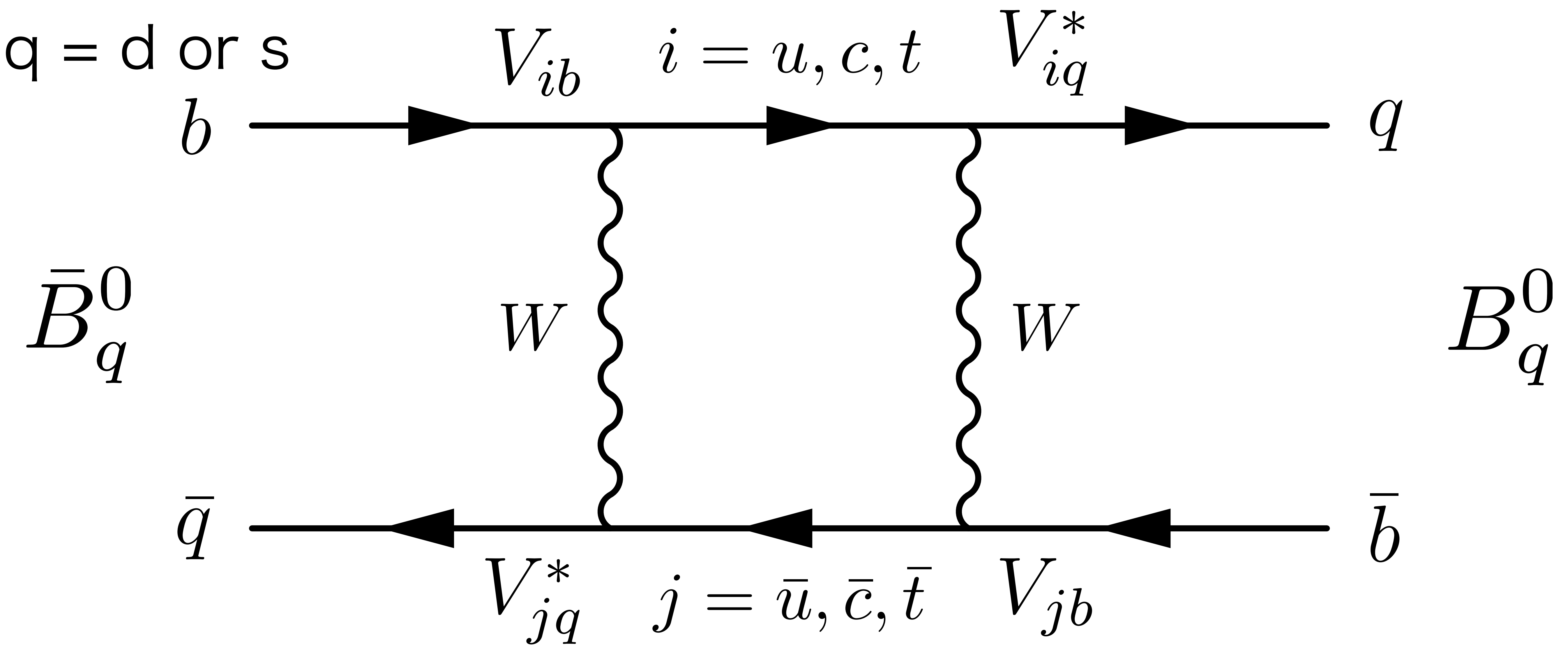
A.J. Bevan et al. "The Physics of the B factories", EPJC74(2014)3026.

4. CP violation in B decays

B–$\bar{B}$ mixing in the standard model

box diagram

q = d or s



Wolfenstein parametrization

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix}$$

$$\lambda \simeq 0.22\,,\ \ A, \rho, \eta \sim O(1)$$

topの寄与が支配的

$$M_{12}(B_d) \propto (V_{tb}V_{td}^*)^2 \quad \sim O(\lambda^6)\ \text{complex}$$

$$M_{12}(B_s) \propto (V_{tb}V_{ts}^*)^2 \quad \sim O(\lambda^4)\ \text{almost real}$$

Time-dependent CP asymmetry

$$\Gamma(B^0(t) \to f) \propto 1 + |\lambda_f|^2 + (1 - |\lambda_f|^2)\cos\Delta m\, t + 2\,\mathrm{Im}\lambda_f \sin\Delta m\, t$$

$$\Gamma(\bar{B}^0(t) \to f) \propto 1 + |\lambda_f|^2 - (1 - |\lambda_f|^2)\cos\Delta m\, t - 2\,\mathrm{Im}\lambda_f \sin\Delta m\, t$$

$$\lambda_f = \frac{q}{p}\frac{\langle f|\bar{B}^0\rangle}{\langle f|B^0\rangle} \simeq \frac{M_{12}^*}{|M_{12}|}\frac{\langle f|\bar{B}^0\rangle}{\langle f|B^0\rangle}$$

$$\begin{aligned}\mathcal{A}_f &= \frac{\Gamma(\bar{B}^0(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\bar{B}^0(t) \to f) + \Gamma(B^0(t) \to f)} \\ &= S_f \sin\Delta m\, t - C_f \cos\Delta m\, t\end{aligned}$$

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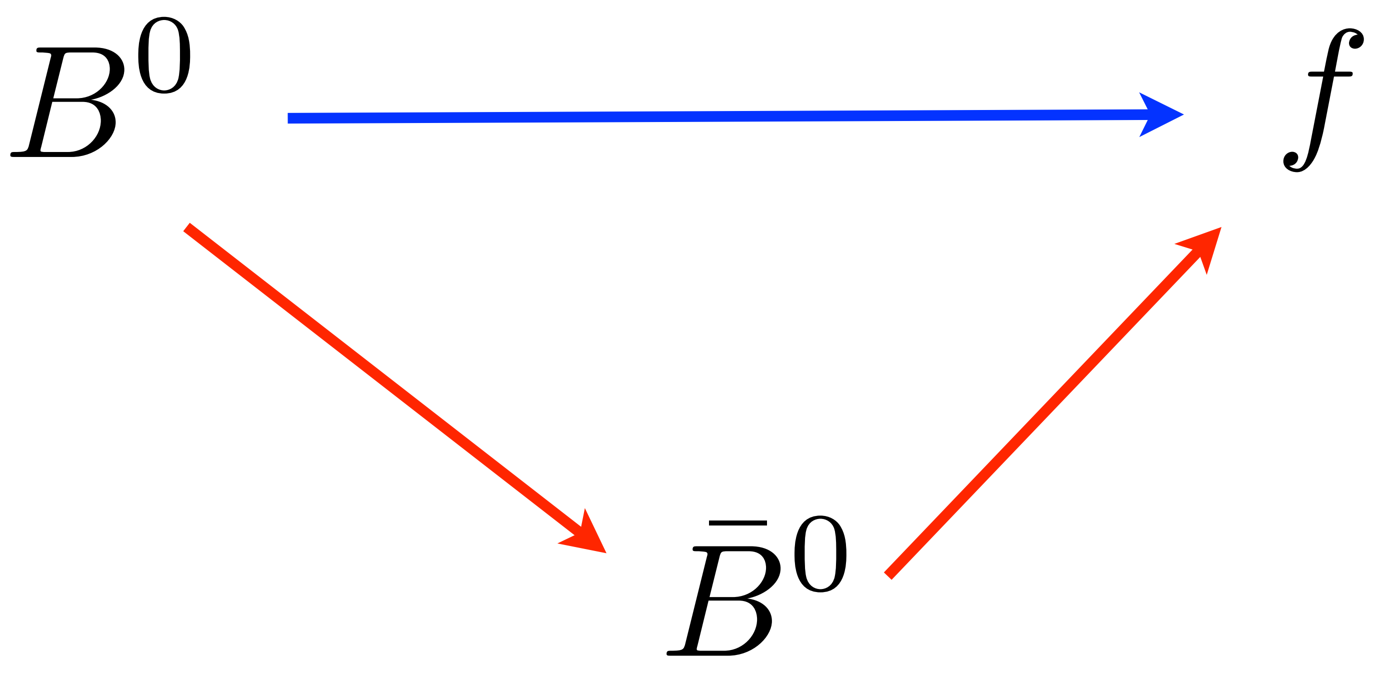
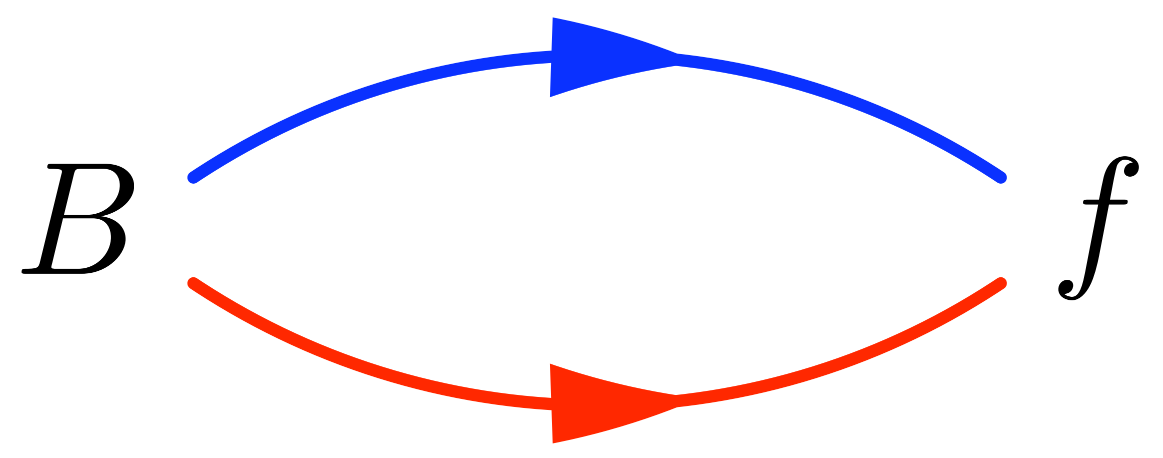
$$S_f = -\frac{2\,\mathrm{Im}\lambda_f}{1+|\lambda_f|^2}$$

Mixing-induced CPV



$$C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}$$

Direct CPV



$$B_d \to J/\psi\, K_S$$

$$\lambda_{J/\psi K_S} = \frac{M_{12}^*}{|M_{12}|} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} = e^{-2i\phi_1}$$

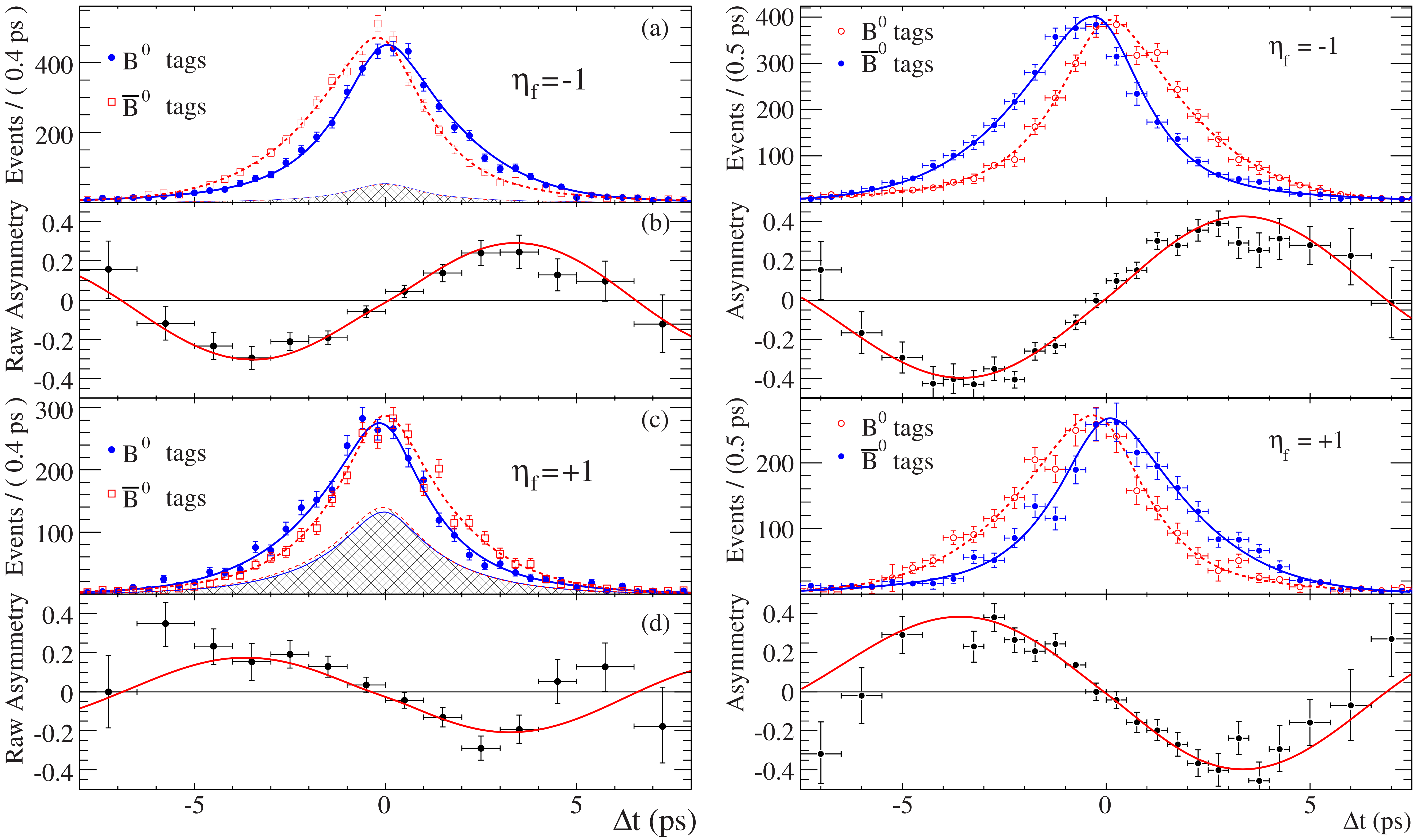
$$S_{J/\psi K_S} = \sin 2\phi_1$$

$$C_{J/\psi K_S} = 0$$

BaBar   Belle

K_S

K_L



A.J. Bevan et al.
"The Physics of the B factories",
EPJC74(2014)3026.

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5. Summary

- Quark flavor mixing and CP violation in the SM
  CKM matrix: 3 mixing angles and 1 phase, UT
- Meson-antimeson mixing
  Degenerate two-level open quantum system
- CP violation in B decays
  Time-dependent CP asymmetry $B_d \to J/\psi\, K_S$
- Subjects not explained
  $|V_{ij}|, \phi_{2,3}$ determination, HQET, EFT and renormalization, rare decays, LFU, (c)LFV, neutrino physics, etc.