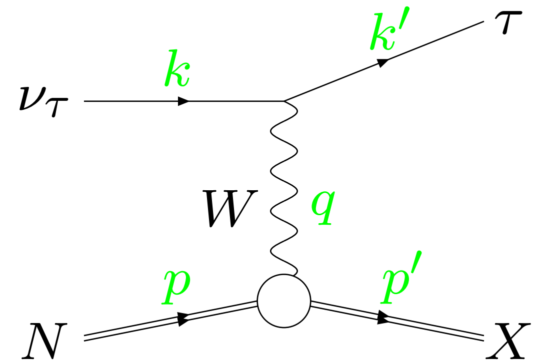
## 3.1 Kinematics and Formalism

$$\boxed{\nu_\tau/\bar{\nu}_\tau(k) + N(p) \longrightarrow \tau^-/\tau^+(k') + X(p')}$$




define 4-momenta in the Lab. frame and Lorentz invariant variables

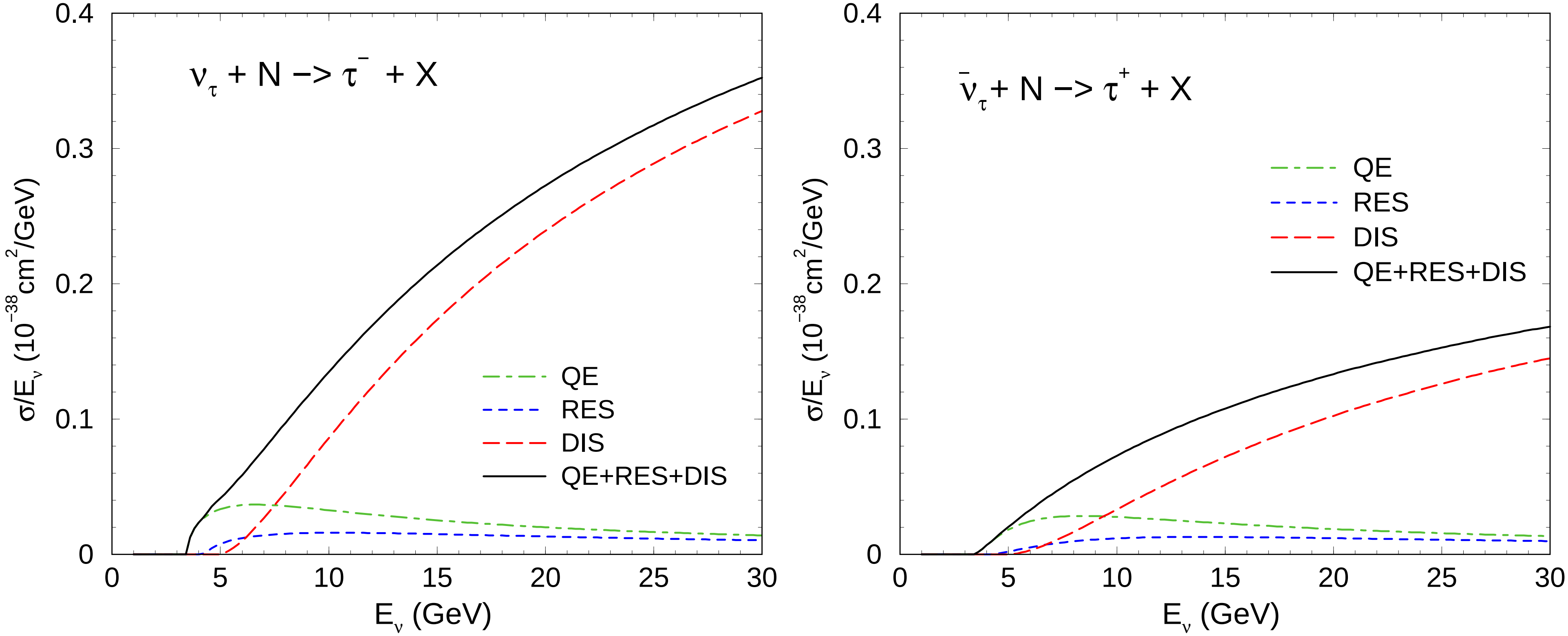
$$\left\{\begin{array}{l} k^\mu = (E_\nu,\ 0,\ 0,\ E_\nu) \\ p^\mu = (M,\ 0,\ 0,\ 0) \\ k'^\mu = (E_\tau,\ p_\tau \sin\theta,\ 0,\ p_\tau \cos\theta) \\ \qquad (p_\tau = \sqrt{E_\tau^2 - m_\tau^2}) \end{array}\right. \qquad \left\{\begin{array}{l} Q^2 = -q^2, \quad q^\mu = k^\mu - k'^\mu \\ W^2 = (p+q)^2 = p'^2 \\ x = \dfrac{Q^2}{2p\cdot q} = \dfrac{Q^2}{2M(E_\nu - E_\tau)} \\ y = \dfrac{p\cdot q}{p\cdot k} = 1 - \dfrac{E_\tau}{E_\nu} \end{array}\right.$$

◇ The kinematical region

We divide the kinematical region into three parts depending on the hadronic invariant mass $W$.

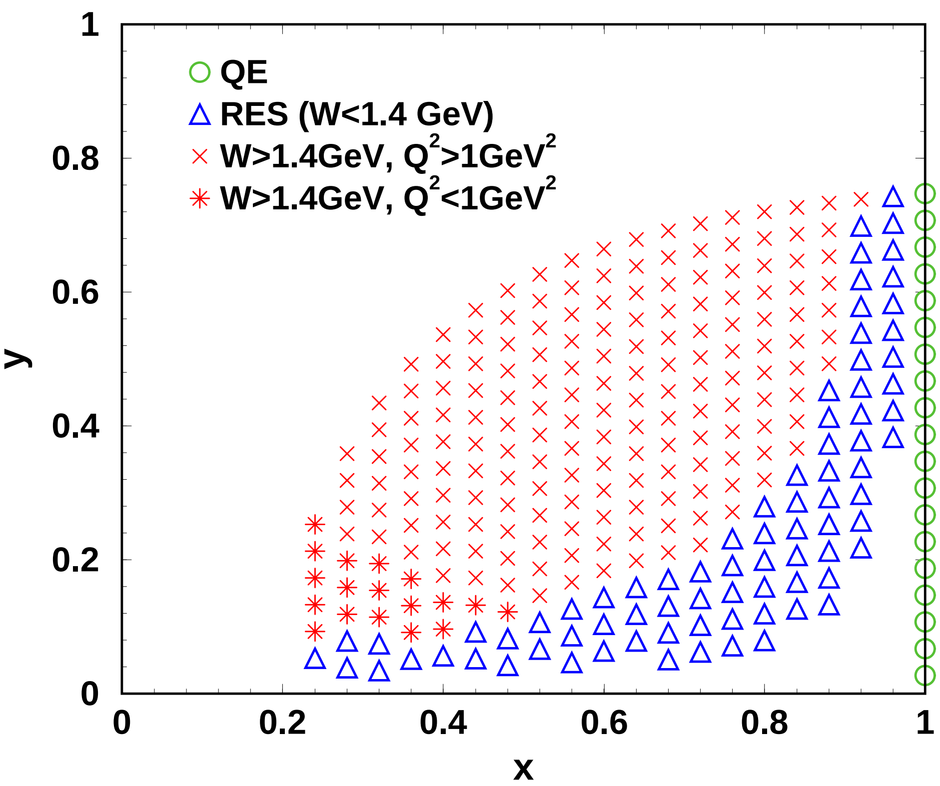
$$W = \left\{\begin{array}{lll} M & \cdots & \text{QE (Quasi – Elastic Scattering)} \\ M + M_\pi \sim 1.4\text{GeV} & \cdots & \text{RES (Resonance Production)} \\ \Downarrow & \cdots & \text{DIS (Deep Inelastic Scattering)} \end{array}\right.$$

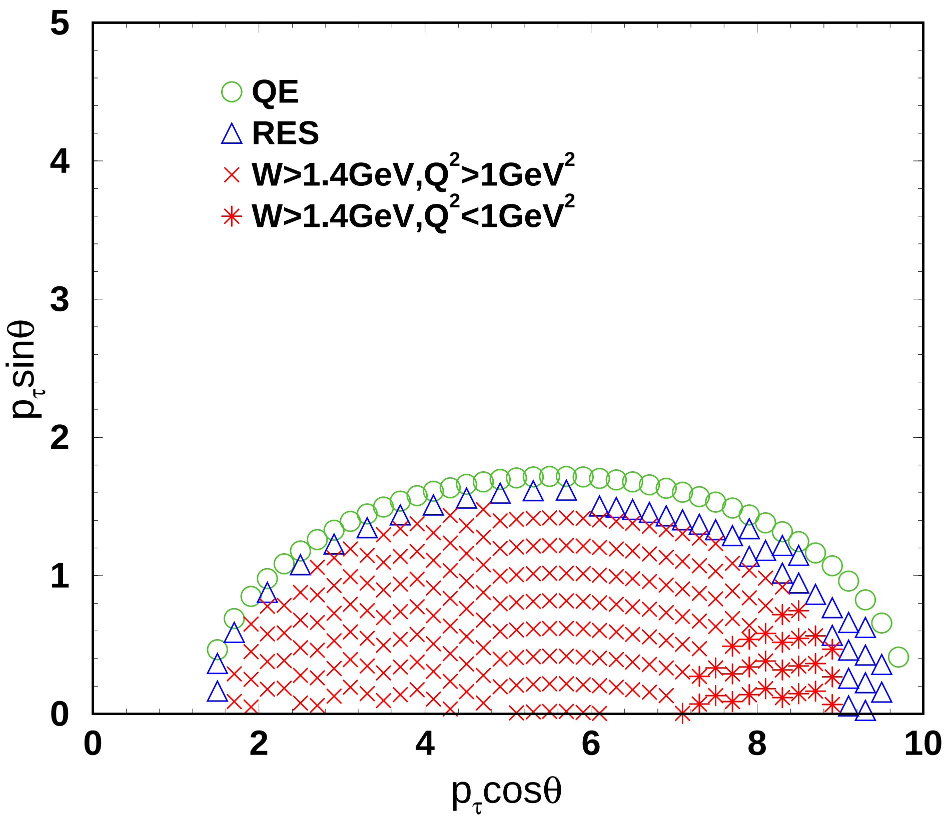
## ◇ The total cross seciton



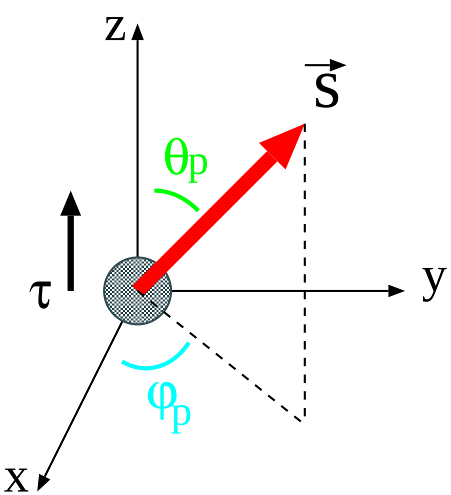
◇ The kinematical regions

$$(E_\nu = 10\mathrm{GeV})$$



## ◇ The spin polarization of $\tau$




define the spin polarization vector of $\tau$

$$\vec{s} = \frac{P}{2}(\sin\theta_P \cos\varphi_P,\ \sin\theta_P \sin\varphi_P,\ \cos\theta_P)$$

$$\left[ P \ : \text{ the degree of polarization } \begin{cases} P = 1 & \text{fully polarized} \\ P = 0 & \text{fully unpolarized} \end{cases} \right]$$

The spin polarization vector is related with the spin density matrix $R_{\lambda\lambda'}$ ($\lambda$ : $\tau$ helicity in the Lab. frame):

$$\frac{dR_{\lambda\lambda'}}{dE_\tau\, d\cos\theta} = \frac{1}{2}\begin{pmatrix} 1 + P\cos\theta_P & P\sin\theta_P\, e^{-i\varphi_P} \\ P\sin\theta_P\, e^{i\varphi_P} & 1 - P\cos\theta_P \end{pmatrix} \frac{d\sigma_{\text{sum}}}{dE_\tau\, d\cos\theta}$$

The density matrix could be calculated as

$$R_{\lambda\lambda'} \propto \sum M_\lambda M^*_{\lambda'} = L^{\mu\nu}_{\lambda\lambda'} W_{\mu\nu} \quad (d\sigma_{\text{sum}} = \text{Tr}[R_{\lambda\lambda'}] = dR_{++} + dR_{--}).$$

## ◇ The spin density matrix in the Lab. frame

$$\frac{dR_{\lambda\lambda'}}{dE_\tau\, d\cos\theta} = \frac{G_F^2}{4\pi M}\frac{p_\tau}{E_\nu} L^{\mu\nu}_{\lambda\lambda'} W_{\mu\nu}^{(\mathrm{QE,RES,DIS})} \qquad (G_F : \text{Fermi constant})$$

## ◇ The leptonic tensor

- for $\tau^-$ production, $L^{\mu\nu}_{\lambda\lambda'} = j^\mu_\lambda\, j^{*\nu}_{\lambda'}$
  (the leptonic weak current)

$$j^\mu_\lambda = \bar{u}_\tau(k',\lambda)\,\gamma^\mu \frac{1-\gamma_5}{2}\, u_\nu(k)$$
$$= \begin{cases} \sqrt{2E_\nu(E_\tau - p_\tau)}\, \left(\sin\frac{\theta}{2},\, -\cos\frac{\theta}{2},\, i\cos\frac{\theta}{2},\, \sin\frac{\theta}{2}\right) & (\lambda = +) \\ \sqrt{2E_\nu(E_\tau + p_\tau)}\, \left(\cos\frac{\theta}{2},\, \sin\frac{\theta}{2},\, -i\sin\frac{\theta}{2},\, \cos\frac{\theta}{2}\right) & (\lambda = -) \end{cases}$$

- for $\tau^+$ production, $\overline{L}^{\mu\nu}_{\lambda\lambda'} = \bar{j}^\mu_\lambda\, \bar{j}^{*\nu}_{\lambda'}$

$$\bar{j}^\mu_\lambda = \bar{v}_\nu(k)\,\gamma^\mu \frac{1-\gamma_5}{2}\, v_\tau(k',\lambda)$$
$$= \begin{cases} \sqrt{2E_\nu(E_\tau + p_\tau)}\, \left(\cos\frac{\theta}{2},\, \sin\frac{\theta}{2},\, i\sin\frac{\theta}{2},\, \cos\frac{\theta}{2}\right) & (\lambda = +) \\ \sqrt{2E_\nu(E_\tau - p_\tau)}\, \left(-\sin\frac{\theta}{2},\, \cos\frac{\theta}{2},\, i\cos\frac{\theta}{2},\, -\sin\frac{\theta}{2}\right) & (\lambda = -) \end{cases}$$

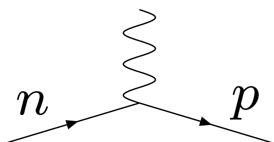
# 3.2 QE (Quasi-Elastic Scattering)

$$\boxed{\nu_\tau + n \to \tau^- + p, \quad \bar{\nu}_\tau + p \to \tau^+ + n}$$

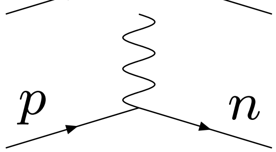
◇ <u>The hadronic tensor (QE)</u>

$$W_{\mu\nu}^{(\mathrm{QE})} = \frac{\cos^2\theta_c}{4} \sum_{\mathrm{spins}} J_\mu^{(\pm)} J_\nu^{(\pm)*} \, \delta(W^2 - M^2) \quad (\theta_c : \text{the Cabibbo angle})$$

(the $p \leftrightarrow n$ transition current)




$$J_\mu^{(+)} = \langle p(p') | \hat{J}_\mu^{(+)} | n(p) \rangle = \bar{u}_p(p') \, \Gamma_\mu \, u_n(p)$$




$$J_\mu^{(-)} = \langle n(p') | \hat{J}_\mu^{(-)} | p(p) \rangle = \langle p(p) | \hat{J}_\mu^{(+)} | n(p') \rangle^*$$

$$\Gamma_\mu = \gamma_\mu \, F_1^V(q^2) + \frac{i\sigma_{\mu\alpha} q^\alpha \xi}{2M} F_2^V(q^2) + \frac{q_\mu}{M} F_3^V(q^2) + \left[ \gamma_\mu \, F_A(q^2) + \frac{(p+p')_\mu}{M} F_3^A(q^2) + \frac{q_\mu}{M} F_p(q^2) \right] \gamma_5$$

($F_{1,2,3}^V$, $F_A$, $F_3^A$ and $F_p$ : form factors, $\quad \xi = \mu_p - \mu_n$)

These form factors are determined theoretically and experimentally.

## 3.3 RES (Resonance Production)

$$\boxed{\nu_\tau + n/p \to \tau^- + \Delta^+/\Delta^{++}, \quad \bar{\nu}_\tau + p/n \to \tau^+ + \Delta^0/\Delta^-}$$

◇ <u>The hadronic tensor (RES)</u>

$$W_{\mu\nu}^{\text{(RES)}} = \frac{\cos^2\theta_c}{4}\sum_{\text{spins}} J_\mu J_\nu^* \frac{1}{\pi}\frac{M_\Delta \Gamma_\Delta}{(W^2 - M_\Delta^2)^2 + M_\Delta^2\Gamma_\Delta^2}$$

(the current in $\nu_\tau + n \to \tau^- + \Delta^+$)

$$J_\mu = \langle \Delta^+ | \hat{J}_\mu | n\rangle = \bar{\psi}^\alpha \,\Gamma_{\mu\alpha}\, u_n \quad (\psi^\alpha : \text{the spin } 3/2 \text{ particle wave function})$$

$$( \, \langle \Delta^{++} | \hat{J}_\mu | p\rangle = \sqrt{3}\langle \Delta^+ | \hat{J}_\mu | n\rangle = \sqrt{3}\langle \Delta^0 | \hat{J}_\mu | p\rangle = \langle \Delta^- | \hat{J}_\mu | n\rangle \, )$$

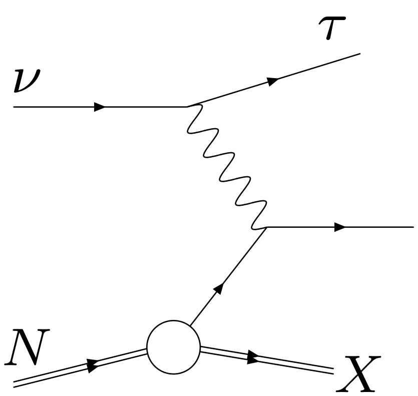
$$\Gamma_{\mu\alpha} = \left[\frac{g_{\mu\alpha}\,\not{q} - \gamma_\mu q_\alpha}{M}C_3^V(q^2) + \frac{g_{\mu\alpha}\,p'\cdot q - p'_\mu q_\alpha}{M^2}C_4^V(q^2) + \frac{g_{\mu\alpha}\,p\cdot q - p_\mu q_\alpha}{M^2}C_5^V(q^2) + \frac{q_\mu q_\alpha}{M^2}C_6^V(q^2)\right]\gamma_5$$
$$+\frac{g_{\mu\alpha}\,\not{q} - \gamma_\mu q_\alpha}{M}C_3^A(q^2) + \frac{g_{\mu\alpha}\,p'\cdot q - p'_\mu q_\alpha}{M^2}C_4^A(q^2) + g_{\mu\alpha}C_5^A(q^2) + \frac{q_\mu q_\alpha}{M^2}C_6^A(q^2)$$

# 3.4 DIS (Deep Inelastic Scattering)

$$\boxed{\nu_\tau + q \to \tau^- + q', \quad \bar{\nu}_\tau + q \to \tau^+ + q'}$$

◇ The hadronic tensor (DIS) using the quark-parton model

$$W_{\mu\nu}^{(\mathrm{DIS})} = \sum_{q,\bar{q}} \int \frac{d\xi}{\xi} f_{q,\bar{q}}(\xi, Q^2)\, K_{\mu\nu}^{(q,\bar{q})}(p_q, q)$$




$\xi$ : the momentum fraction of quark and nucleon $(p_q = \xi p)$

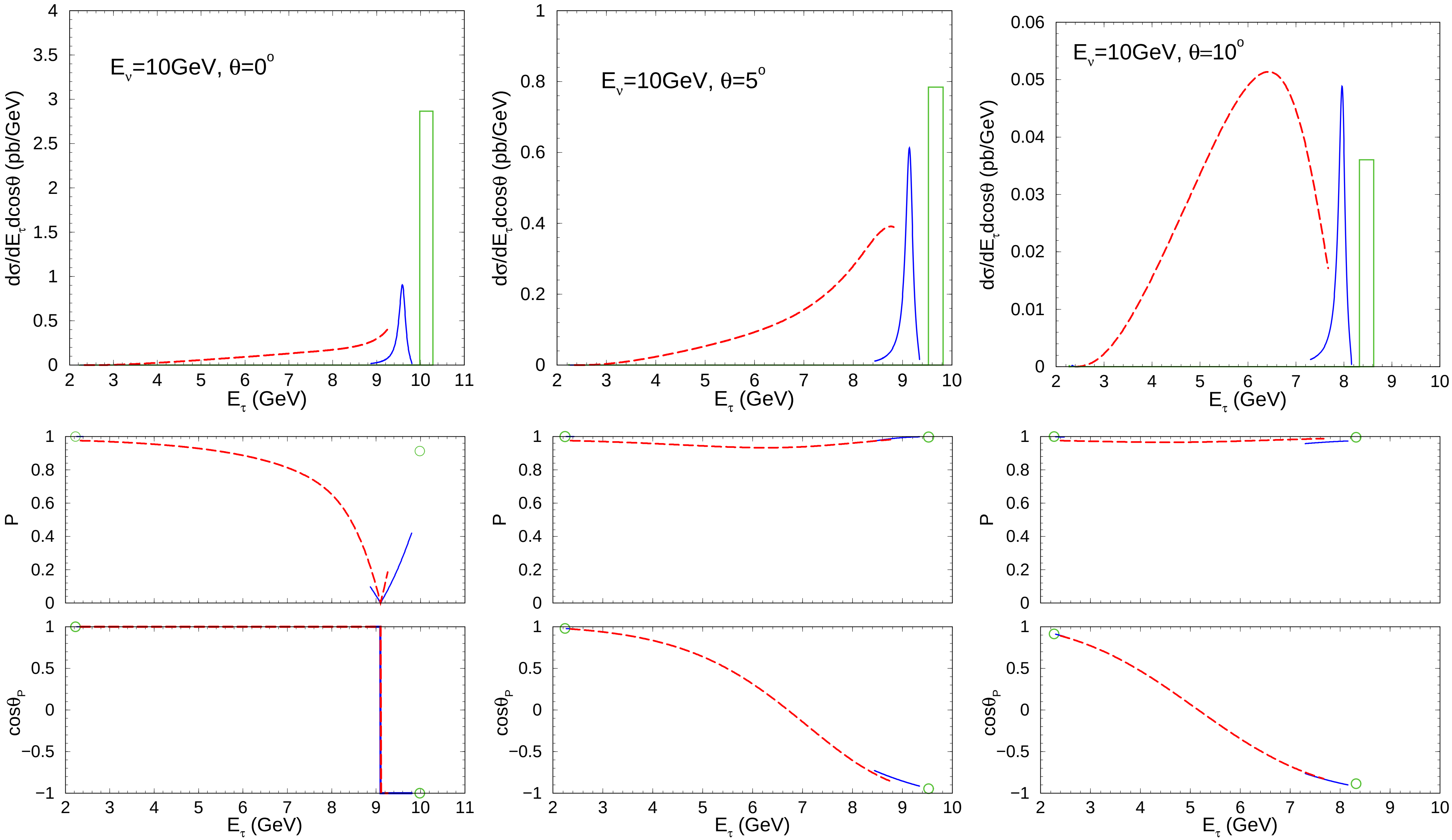
$f_{q,\bar{q}}(\xi, Q^2)$ : the parton distribution functions (PDFs) for each flavor

(the quark tensor)

$$K_{\mu\nu}^{(q,\bar{q})} = \delta(2\, p_q \cdot q - Q^2)$$
$$\times 2 \left[ -g_{\mu\nu}(p_q \cdot q) + 2 p_{q\mu} p_{q\nu} \mp i \epsilon_{\mu\nu\alpha\beta} p_q^\alpha q^\beta + (p_{q\mu} q_\nu + q_\mu p_{q\nu}) \right]$$

# 3.5 Polarization of Produced $\tau$

$$\boxed{\nu_\tau + N \to \tau^- + X \quad E_\nu = 10\text{GeV}}$$



## ◇ Polarization of the produced $\tau^{\pm}$