

Probing new intra-atomic force with isotope shifts

— Implication of precision spectroscopy of 10^{-18} accuracy —

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arXiv: 1710.11443

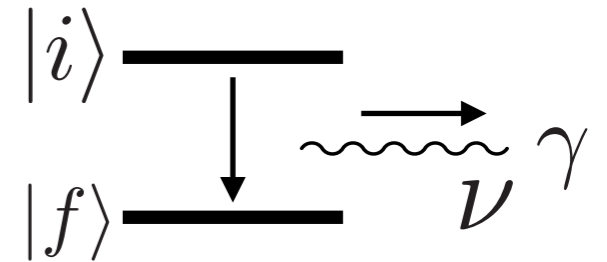
Het Camp, 2-4 Nov. 2017, Nose, Osaka, Japan

Isotope shift (IS)

Transition frequency difference between isotopes

$$h\nu_A = E_A^i - E_A^f$$

$$\text{IS} = \nu_{A'A} := \nu_{A'} - \nu_A$$



No IS for infinitely heavy and point-like nuclei

→ $\text{IS} = \text{MS} + \text{FS}$

Mass shift: finite mass of nuclei (reduced mass)

$$\text{MS} \propto \mu_{A'} - \mu_A \quad (\text{dominant for small } Z)$$

Field shift: finite size of nuclei

$$\text{FS} \propto r_{A'}^2 - r_A^2 \quad (\text{dominant for large } Z)$$

Theoretical calculation of IS: not easy

$$\text{IS} \sim O(\text{GHz}) \sim O(10 \mu\text{eV})$$

King's linearity

King, 1963

IS of two transitions: $\ell = 1, 2$

$$\nu_{A'A}^{\ell} = K_{\ell} \mu_{A'A} + F_{\ell} r_{A'A}^2$$

$$\mu_{A'A} := \mu_{A'} - \mu_A$$

$$r_{A'A}^2 := \langle r^2 \rangle_{A'} - \langle r^2 \rangle_A$$

Modified IS: $\tilde{\nu}_{A'A}^{\ell} := \nu_{A'A}^{\ell} / \mu_{A'A}$

$$\tilde{\nu}_{A'A}^{\ell} = \boxed{K_{\ell}} + \boxed{F_{\ell} r_{A'A}^2 / \mu_{A'A}} \text{ nuclear factor}$$

electronic factors

King's linearity eliminating the nuclear factor

$$\tilde{\nu}_{A'A}^2 = K_{21} + \frac{F_2}{F_1} \tilde{\nu}_{A'A}^1$$

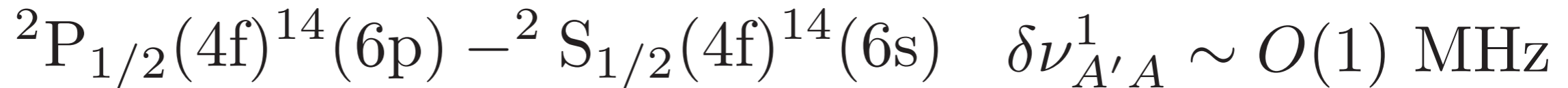
$$K_{21} := K_2 - \frac{F_2}{F_1} K_1$$

→ $(\tilde{\nu}_{A'A}^1, \tilde{\nu}_{A'A}^2)$ on a straight line, King's plot

IS data of Yb⁺

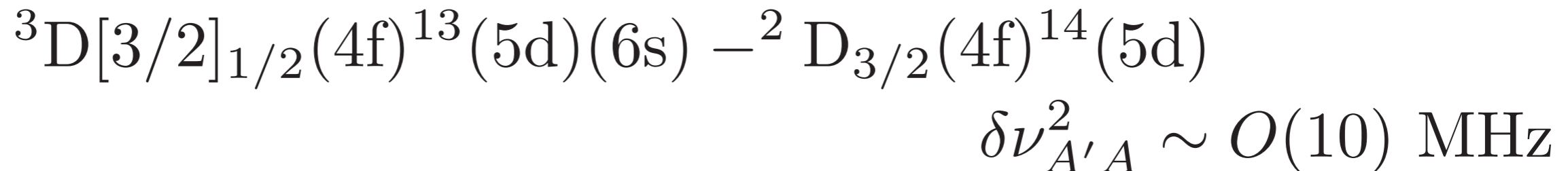
Line 1: 369 nm

Martensson-Pendrill et al. PRA49, 3351 (1994)



Line 2: 935nm

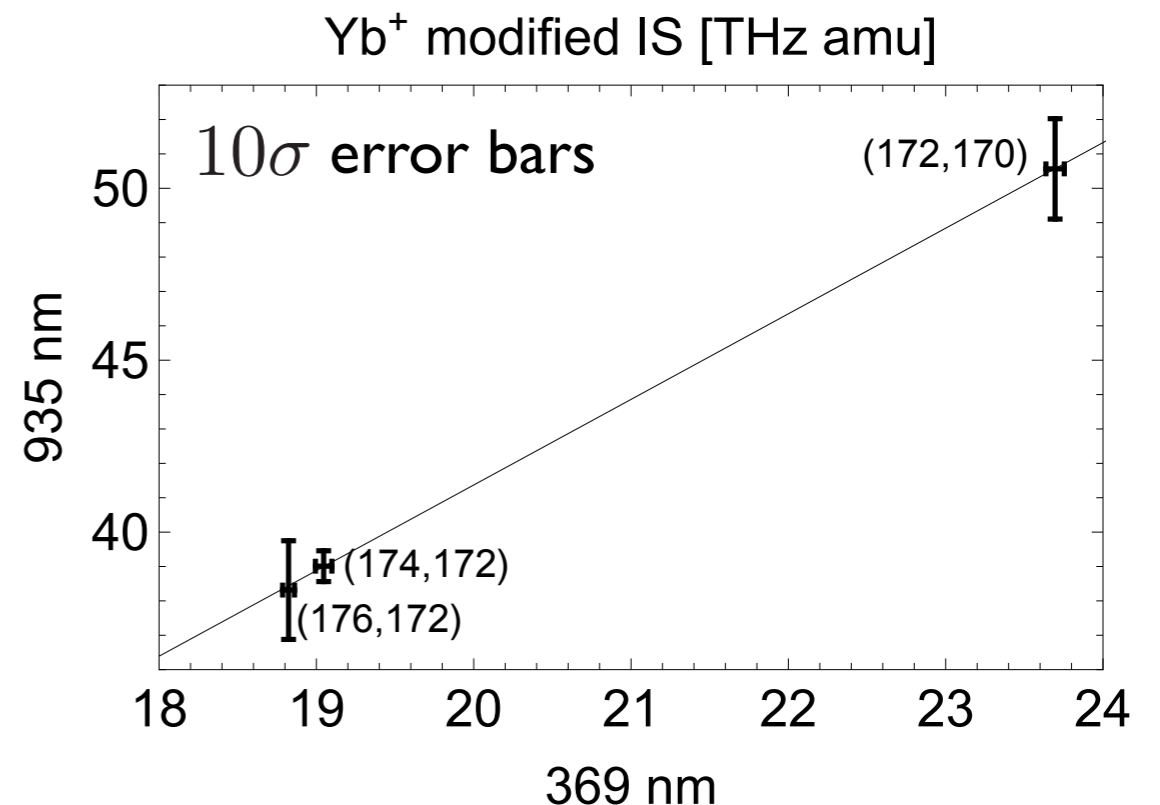
Sugiyama et al. CPEM2000



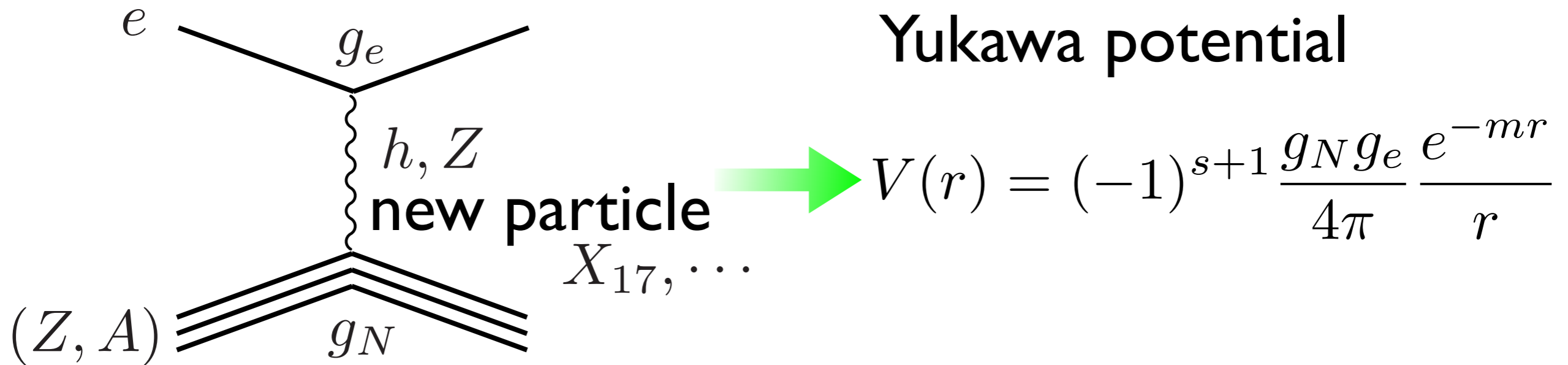
Isotope pairs: (172, 170), (174, 172), (176, 172)

King's plot

linear within errors



Particle shift (PS)



Frequency shifts by particle exchange (Yb⁺ g.s.)

$$|\Delta\nu| \sim \begin{cases} 10^{-4} \text{ Hz} & \text{Higgs (SM)} \\ 400 \text{ Hz} & \text{Higgs (LHC bound)} \\ 800 \text{ Hz} & Z \\ 10 \text{ MHz} & X_{17} \text{ 17 MeV vector boson} \end{cases}$$

<< theoretical uncertainties

Breakdown of the linearity by PS

Delaunay et al. arXiv:1601.05087v2

$$IS = MS + FS + PS$$

PS by new neutron-electron interaction

$$\nu_{A'A}^{\ell} = K_{\ell} \mu_{A'A} + F_{\ell} r_{A'A}^2 + X_{\ell} (A' - A)$$

Generalized King's relation

$$\tilde{\nu}_{A'A}^2 = K_{21} + F_{21} \tilde{\nu}_{A'A}^1 + \varepsilon A'A \quad \text{nonlinearity}$$

probe into new physics

PS nonlinearity

$$\varepsilon_{PS} = X_1 \left(\frac{X_2}{X_1} - \frac{F_2}{F_1} \right) \quad X_{\ell} \propto \frac{g_n g_e}{m^2} \text{ as } m \rightarrow \infty$$

Field shift nonlinearity

One of the sources of nonlinearity in QED

$$\text{FS} = F_\ell r_{A'A}^2 + G_\ell r_{A'A}^4$$

$$\tilde{\nu}_{A'A}^2 = K_{21} + F_{21} \tilde{\nu}_{A'A}^1 + \varepsilon_{A'A}$$

 $\varepsilon = \varepsilon_{\text{PS}} + \varepsilon_{\text{FS}}$

Wavefunction inside the nucleus is relevant.

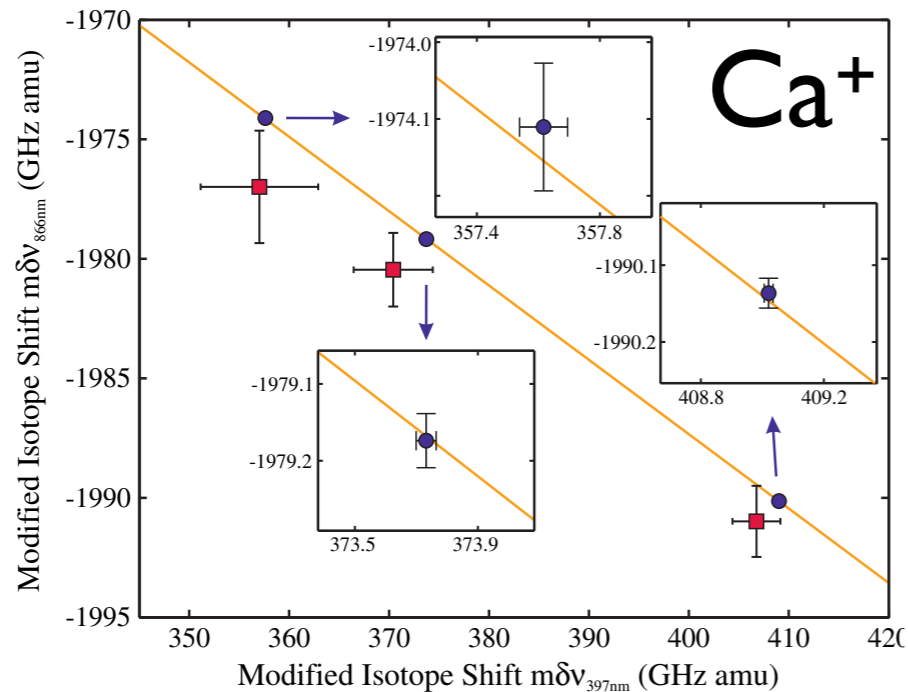
p state dominant: $\text{Ca}^+ 4p, \text{Yb}^+ 6p$

$$\varepsilon_{\text{FS}} = Z |\psi'_{np}(0)|^2 \frac{d}{dA} \langle r^4 \rangle_A + \dots$$

 nuclear Helm distribution

Present constraint and future prospect

Data fitting with $\tilde{\nu}_{A'A}^2 = K_{21} + F_{21}\tilde{\nu}_{A'A}^1 + \varepsilon A'A$

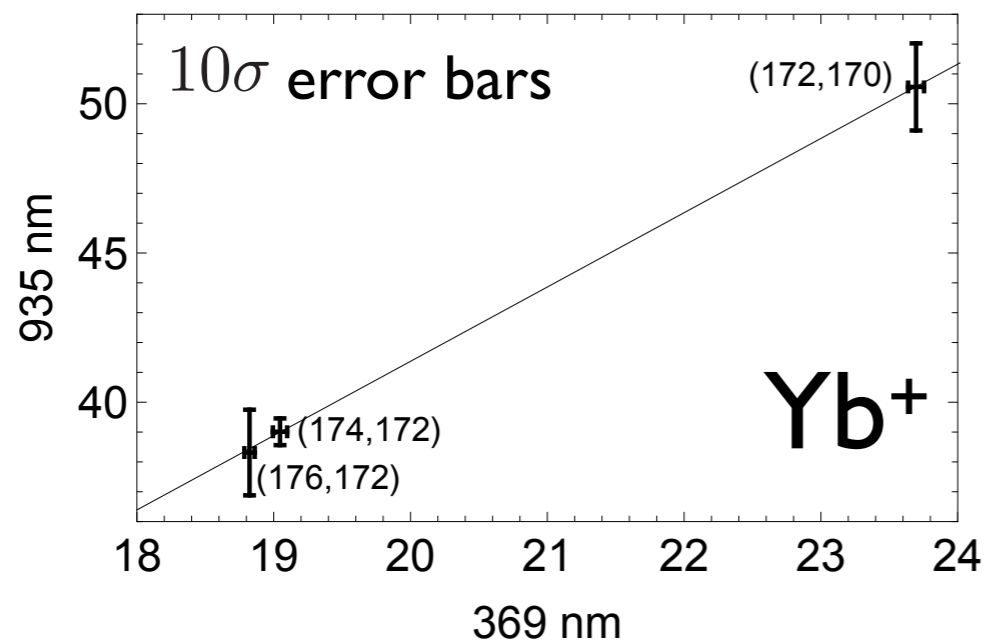


$$\varepsilon = (-2.45 \pm 4.05) \cdot 10^{-6} \text{ au}$$

future prospect $\delta\nu = 1 \text{ Hz}$

$$|\varepsilon| < 4.5 \cdot 10^{-11}$$

Yb⁺ modified IS [THz amu]

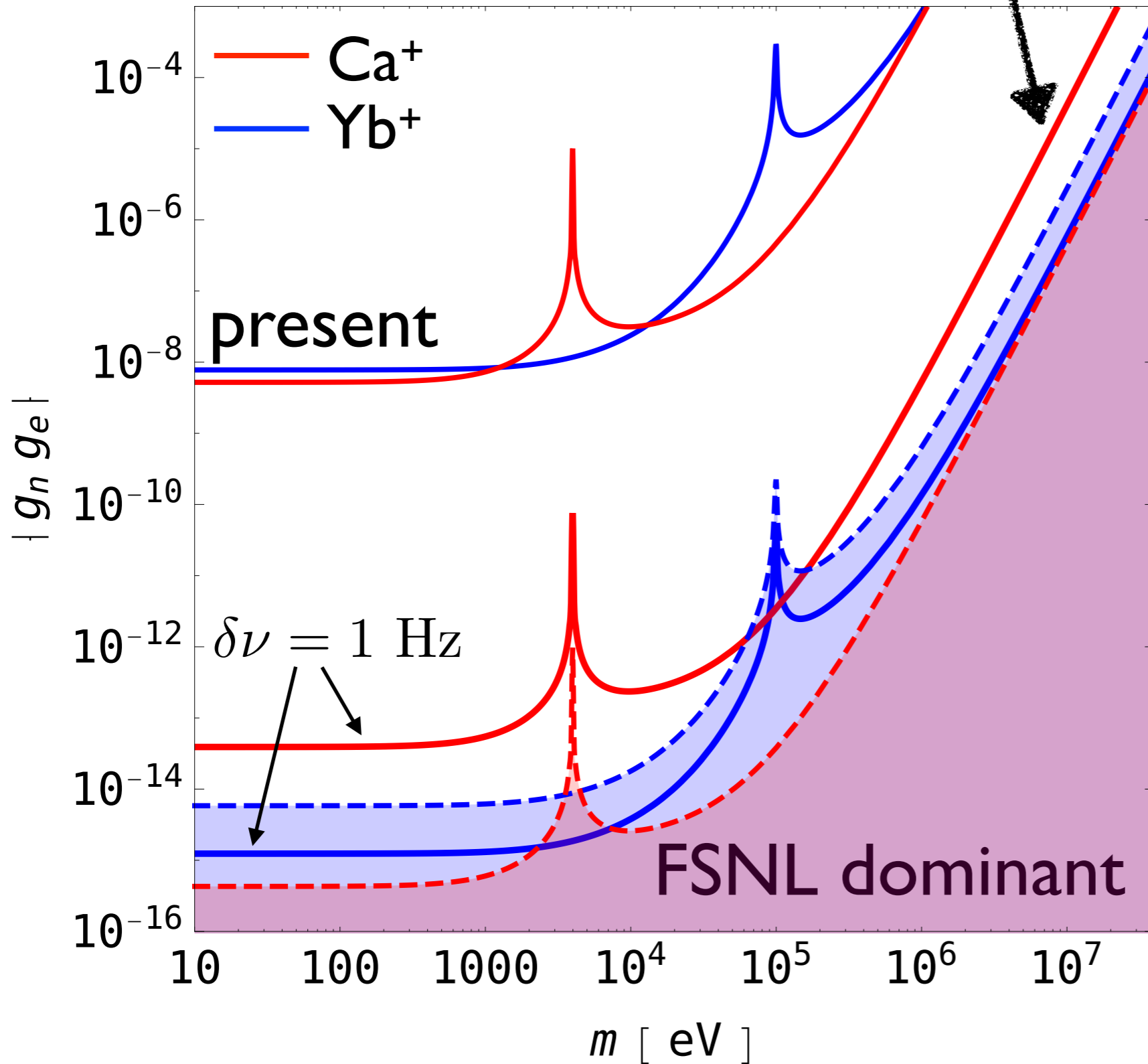


$$\varepsilon = (-1.26 \pm 1.35) \cdot 10^{-4}$$

future prospect $\delta\nu = 1 \text{ Hz}$

$$|\varepsilon| < 4.2 \cdot 10^{-11}$$

$O(m^4)$ due to p states

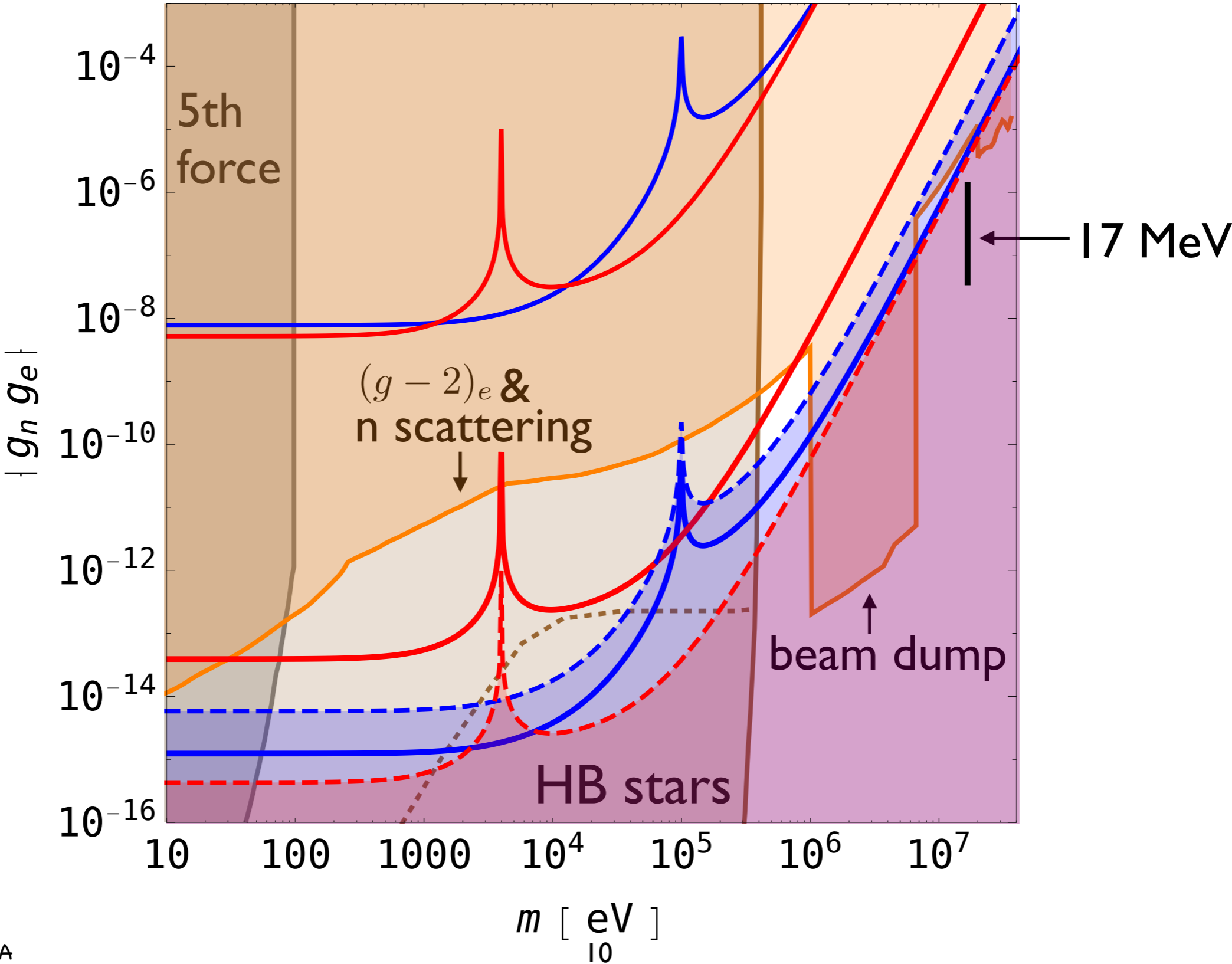


FSNL dominance:

$$\text{Ca}^+ \quad \delta\nu \lesssim 0.01 \text{ Hz}$$

$$\text{Yb}^+ \quad \delta\nu \lesssim 4.7 \text{ Hz}$$

Comparison to other constraints: vector



Summary and outlook

■ Isotope shift and King's linearity

$$\text{IS}=\text{MS}+\text{FS}, \quad \tilde{\nu}_{A'A}^2 = K_{21} + F_{21}\tilde{\nu}_{A'A}^1$$

Linear relation of modified IS of two lines

■ Nonlinearity $\tilde{\nu}_{A'A}^2 = K_{21} + F_{21}\tilde{\nu}_{A'A}^1 + \varepsilon A'A$

$$\varepsilon = \varepsilon_{\text{PS}} + \varepsilon_{\text{FS}}$$

Particle shift nonlinearity: $\varepsilon_{\text{PS}} \sim O(1/m^4)$

sensitive for lighter particles, $m \ll 100 \text{ MeV}$

Other nonlinearities: more study needed

■ Yb⁺ ion trap project by Sugiyama et al. (Kyoto)

$$\delta\nu < 1 \text{ Hz} \sim 100 \text{ kHz}$$

possible with proved technique

Backup

Frontiers in particle physics

Energy frontier: LHC, ILC,...

Intensity frontier: B factory, muon,...

Cosmic frontier: CMB,...

Precision / low energy frontier

$0\nu\beta\beta$, DM, EDM,...

Temporal variation of fundamental constants

α , m_e/m_p using atomic clock

Yb^+ : $\delta\nu/\nu \sim 10^{-18}$, $\delta\nu \sim \text{sub Hz}$

Hunteman et al. (PTB) 2016

Isotope shift new neutron-electron interaction

Heavy particle limit

$$ma_B \gg Z, \quad a_B = \text{Bohr radius} \sim (4 \text{ keV})^{-1}$$

$$F_\ell, X_\ell \propto |\psi_{i_\ell}(0)|^2 - |\psi_{f_\ell}(0)|^2 \longrightarrow \lim_{m \rightarrow \infty} \left(\frac{X_2}{X_1} - \frac{F_2}{F_1} \right) = 0$$

Asymptotic behavior of PS

$$\int d^3r |\psi(r)|^2 \frac{e^{-mr}}{r} = \frac{1}{m^2} \sum_{k=0} (2 + 2l + k)! \frac{\xi_k^l}{m^{2l+k}} + \dots$$

$l = \text{angular momentum}$

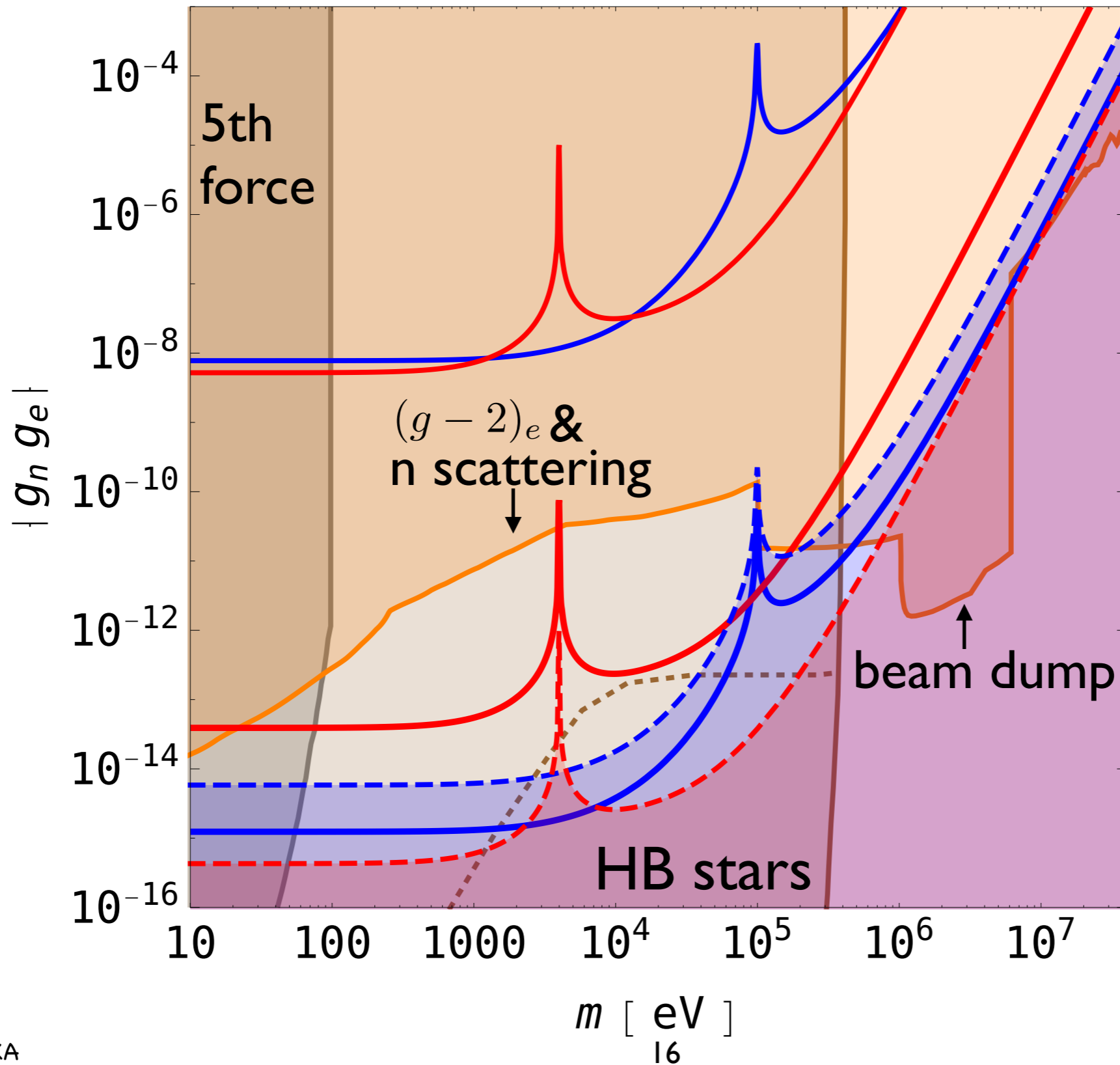
$\xi_1^0 = 0$ for nucl. charge distribution without cusp

$$\frac{X_2}{X_1} - \frac{F_2}{F_1} \sim O\left(\frac{1}{m^2}\right) \longrightarrow \epsilon_{\text{PS}} \sim O\left(\frac{1}{m^4}\right)$$

less sensitive to heavier particles

cf. Berengut et al. arXiv:1704.05068 $\epsilon_{\text{PS}} \propto 1/m^3$

Comparison to other constraints: scalar



^8Be anomaly and 17 MeV vector boson

Krasznahorkay et al. PRL 116, 042501 (2016)



Bump in the $e^+ e^-$ inv. mass

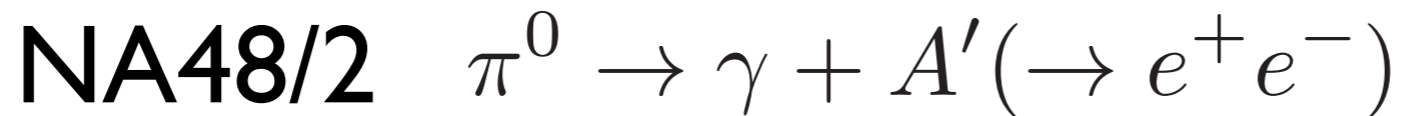


$m_X \sim 17 \text{ MeV}$

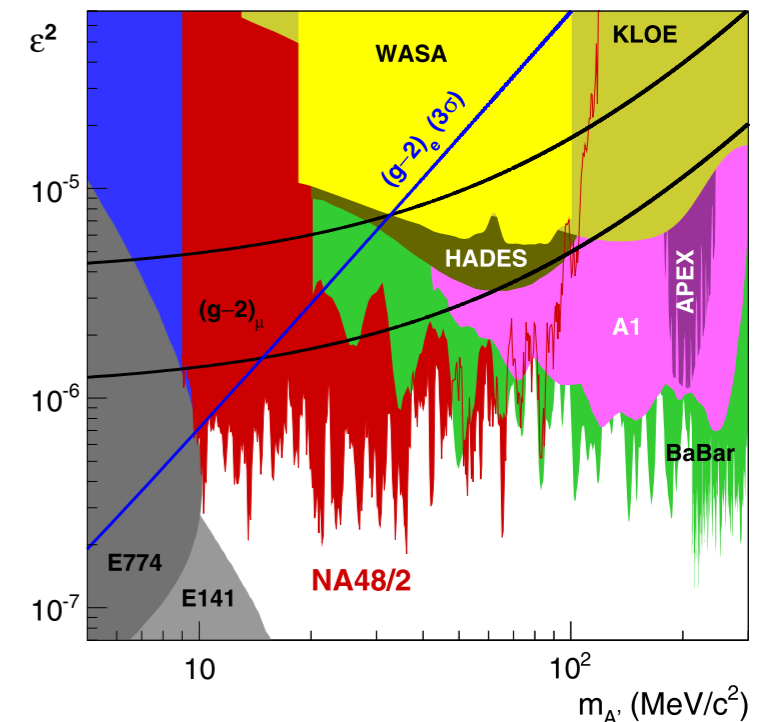
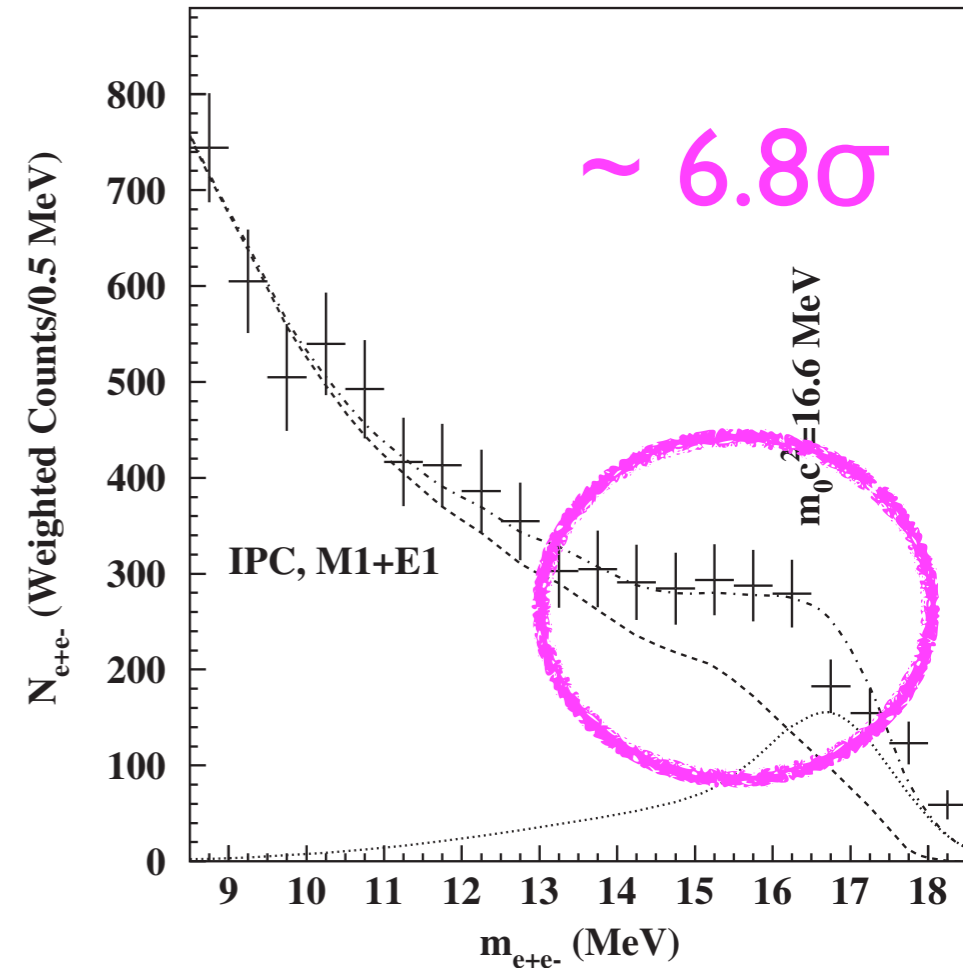
vector $U(1)_B, U(1)_{B-L}$

Constraint from
dark photon search

Feng et al. PRL 117, 071803 (2016)



→ **protophobic**



Evaluation of PS nonlinearity

Single electron approximation

$$X_\ell = \frac{g_n g_e}{4\pi} \int r^2 dr \frac{e^{-mr}}{r} [R_{i_\ell}^2(r) - R_{f_\ell}^2(r)]$$

Wavefunction

non relativistic (not bad for $m \ll 100$ MeV)

Thomas-Fermi model

semiclassical, statistical, selfconsistent field

exact in large Z limit