Deep Learning, Black Hole and Chaos

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AdS/CFT
Quantum gravity
Deep learning
Neuroscience
Chaos
1. Significance from similarity
2. Deep learning: review
3. Scalar field in AdS BH
4. AdS/DL correspondence
5. Emergent space
6. AdS BH is learned
7. Emergent from experiment
Deep Learning

AdS/CFT
[Maldacena ‘97]

https://www.edureka.co/blog/what-is-deep-learning
Significance from similarity

**AdS/CFT**
We don’t know which QFT has a gravity dual.
From a given data, extra dimension emerges?

**Deep learning**
We don’t know continuum limit of deep layers.
From a given data, extra dimension emerges?
Deep learning: optimized sequential map

Layer 1

Layer 2

Layer N

1) Prepare many sets \( \{x_i^{(1)}, F\} \)
2) Let the network learn them by adjusting \( W_{ij} \)
Scalar field in AdS BH

1) Consider a scalar field theory in a curved geometry

\[ S = \int d^{d+1}x \sqrt{-\det g} \left[ (\partial_\eta \phi)^2 - V(\phi) \right] \]

\[ ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2) \]

\[
\begin{aligned}
\text{AdS boundary (} \eta \sim \infty \text{)}: & \quad f \sim g \sim \exp[2\eta/L] \\
\text{Black hole horizon (} \eta \sim 0 \text{)}: & \quad f \sim \eta^2, \ g \sim \text{const.}
\end{aligned}
\]

2) Derive equation of motion and boundary condition

\[ \partial_\eta^2 \phi + h(\eta) \partial_\eta \phi - \frac{\delta V[\phi]}{\delta \phi} = 0, \quad \partial_\eta \phi \bigg|_{\eta=0} = 0 \]

\[ h(\eta) \equiv \partial_\eta \left[ \log \sqrt{f(\eta)g(\eta)^{d-1}} \right] \]
3) Discretize it, Go to Hamilton formulation

\[
\begin{align*}
\phi(\eta + \Delta \eta) &= \phi(\eta) + \Delta \eta \pi(\eta) \\
\pi(\eta + \Delta \eta) &= \pi(\eta) + \Delta \eta \left( h(\eta) \pi(\eta) - \frac{\delta V(\phi(\eta))}{\delta \phi(\eta)} \right)
\end{align*}
\]

4) Map it to deep neural network

\[
\begin{array}{c}
\phi \\
\pi
\end{array}
\begin{array}{c}
\bullet \\
\bullet \\
\bullet
\end{array}
\begin{array}{c}
\pi(\eta = 0)
\end{array}
\]

\[
\begin{array}{c}
\eta = \infty \\
\eta \\
\eta = 0
\end{array}
\]

Dictionary

- Input data: \((\phi(\eta = \infty), \pi(\eta = \infty))\) Order / external field
- Output data: horizon boundary condition \(\pi(\eta = 0) = 0\)
- Weight function \(W^{(a)}_{ij}\): metric \(h(\eta)\)
Emergent space in deep learning

Experiment 1: “Can AdS BH metric be learned?”

1) Use AdS BH metric and generate input data.
2) Prepare network with unspecified metric.
3) Let the network learn it by the data.
4) Check if continuous space emerges.

Experiment 2: “Emergent space from experiments?”

1) Use experimental data.
   Ex) Magnetization curve of strongly correlated material
2) ~ 4) (same as above.)
1: AdS Black hole is learned

Before

After
2: Emergent space from experiments

After
1. Brief history
2. Chaos: review
3. Quantum problem
4. SYK model is dual to BH
5. BH is a nest of chaos
6. Wilson loop is chaotic
Information Preservation and Weather Forecasting for Black Holes

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Abstract

It has been suggested [1] that the resolution of the information paradox for evaporating black holes is that the holes are surrounded by firewalls, bolts of outgoing radiation that would destroy any infalling observer. Such firewalls would break the CPT invariance of quantum gravity and seem to be ruled out on other grounds. A different resolution of the paradox is proposed, namely that gravitational collapse produces apparent horizons but no event horizons behind which information is lost. This proposal is supported by ADS-CFT and is the only resolution of the paradox compatible with CPT. The collapse to form a black hole will in general be chaotic and the dual CFT on the boundary of ADS will be turbulent. Thus, like weather forecasting on Earth, information will effectively be lost, although there would be no loss of unitarity.
2-1 Brief history of quantum gravity/chaos


1997 Maldacena: Discovery of AdS/CFT.
A quantum gravity is nonperturbatively defined.

2008 Sekino-Susskind:
“Black holes are the fastest scramblers”


2014 Kitaev: SYK model saturates the bound.
Chaos: sensitive to initial conditions

Classical chaos
= Non-periodic bounded orbits sensitive to initial conditions in non-linear deterministic dynamical systems

Lyapunov exponent, positive

\[ L = \lim_{t \to \infty} \lim_{d(0) \to 0} \frac{1}{t} \log \frac{d(t)}{d(0)} \]

Poincare section, scattered

\[ d(t) \sim d(0) \exp[Lt] \]
Quantum Problem: Lyapunov washed out?

Our solution: Employing large N limit

Another solution: Out-of-Time-Ordered correlators

\[ \left\langle Q(t)P(0)Q(t)P(0) \right\rangle \sim \left( \frac{\delta Q(t)}{\delta Q(0)} \right)^2 \]

[Larkin, Ovchinnikov `69] [Kitaev `14] [Maldacena, Shenker, Stanford `15]
Lyapunov upper bound (conjecture) for thermal OTO

\[ L \leq 2\pi T \]

[Sachdev, Ye `95, Kitaev `15]

Suggested from AdS/CFT with black holes

SYK (Sachdev-Ye-Kitaev) model  [Kitaev `15][Sachdev,Ye `95]

(1+0 dim., N Majorana fermions, disordered interaction)

\[
H = \frac{-1}{4!} \sum_{i,j,k,l=1}^{N} j_{ijkl} \psi_i \psi_j \psi_k \psi_l
\]

\[
\left( \sum_{j,k,l=1}^{N} \langle j_{ijkl} j_{ijkl} \rangle = 6J^2 \right)
\]

Solvable at Large N and strong coupling  \( \beta J \to \infty \)

Shown to saturate the bound  [Kitaev `15] [Maldacena, Stanford `16]

See for modified models [Gross, Rosenhaus `16] [Witten `16]
Black hole is a nest of chaos

Black hole is the fastest scrambler? [Sekino, Susskind `08]

Shock wave delay [Shenker, Stanford `13, `14]

2d dilaton gravity dual to SYK [Almheiri, Polchinski `14] [Engelsoy, Martens, Verlinde `16]

\[ \delta t_2 = \frac{\delta E}{8\pi TM} e^{2\pi T(t_2 - t_1)} \]

Lyapunov exponent = Hawking temperature
Wilson loop is chaotic

[Murata, Tanahashi, KH, to appear]

\[
S = \mathcal{T} \frac{R^8}{2r_H^5 L^2} \left[ \left( \frac{d}{dt} \delta r \right)^2 + \left(2\pi T_H \right)^2 (\delta r)^2 \right]
\]

Pot. separatrix

Rectangular string model
Wilson loop chaos satisfies the bound

[Murata, Tanahashi, KH, to appear]

Difference of interquark forces exponentially grow
AdS/CFT
Quantum gravity

Deep learning
Neuroscience

Chaos