Spacetime Symmetry
and Nambu-Goldstone Bosons

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with Y. Hidaka (RIKEN) and G. Shiu (Wisconsin&HKIAS)

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1. Introduction
symmetry breaking in physics
spacetime symmetry breaking

cosmology

time-translation

condensed matter

Poincare symmetry
Nambu-Goldstone (NG) modes
Nambu-Goldstone theorem

breaking of continuum symmetry

gapless mode (NG mode)

nonlinear rep of broken sym

# internal symmetry in relativistic system

one broken symmetry $\rightarrow$ one gapless mode with dispersion relation $\omega = k$

ex. pions as pseudo Nambu-Goldstone bosons

cf. recent progress in nonrelativistic case $\rightarrow$ Watanabe-san’s talk
there is yet another story
for spacetime symmetry breaking
mismatch in NG mode counting

ex. 1 string [Low-Manohar ’02]

broken symmetries:
rotation & translation
※ only one NG mode
(string position)

ex. 2 conformal symmetry breaking

conformal symmetry
\[ P_\mu, L_{\mu\nu}, D, K_\mu \]
\[ D \]: dilatation
\[ K_\mu \]: special conformal

Poincare symmetry
\[ P_\mu, L_{\mu\nu}, D, K_\mu \]
only one NG mode:
dilaton (for dilatation)
massive modes for broken symmetries

ex. smectic A phase of liquid crystals

- translation and rotational symmetries are broken

  NG mode $\pi$ for translation: position of layers

  NG mode $\xi_i$ for rotations: rotations of molecules

  \[(\hat{i} = 1, 2)\]

- $\pi$ is massless because of original translation symmetry

  ※ invariance under $\pi \rightarrow \pi + \text{const.}$.

- $\xi_i$ is massive

  ※ original rotation symmetry transforms $\pi$ also

  \[\xi_1 \rightarrow \xi_1 + \omega_{13}, \pi \rightarrow \pi - \omega_{13} x \ (\omega_{13} : \text{const.})\]
“NG mode” for broken spacetime symmetry

massless

massive

no corresponding dof
(unphysical, redundant)

subtleties even in identification of excitations
for broken spacetime symmetries
“NG mode” for broken spacetime symmetry

massless

massive

no corresponding dof
(unphysical, redundant)

in the low-energy effective theory...
“NG mode” for broken spacetime symmetry

in the low-energy effective theory...

massless

$m \lesssim E$

no corresponding dof (unphysical, redundant)
“NG mode” for broken spacetime symmetry

important to understand what is physical, what is massive, ...

massless

$m \lesssim E$

no corresponding dof (unphysical, redundant)

in the low-energy effective theory...
main message: gauging is useful!

global symmetry
translation, Lorentz, conformal, ...

gauge!

local symmetry
diffs, local Lorentz, local Weyl, ...

- identification of physical excitations for broken symmetries

what I discuss today:

- global vs local viewpoints of spacetime symmetry
- EFT from local symmetry point of view
- revisit coset construction based on global symmetry breaking
plan of my talk:

1. Introduction ✔
2. Global vs local
3. EFT from gauging
4. Prospects
2. Global symmetry vs local symmetry

Q. why

# of broken symmetries ≠ # of physical excitations

in a nonlinear rep

in spacetime symmetry case?
scalar vs nonzero spin brane

- same global symmetry breaking pattern:
  translation in z-direction & rotation in x-z, y-z surfaces
- different excitations associated with symmetry breaking
  in nonlinear rep of broken symmetries
ex. rotation = coord transf + local rotation

global rotation around origin = local translation + local rotation
acts on all the local field nonzero spin fields
rotation symmetry breaking

global rotation around origin

= local translation + local rotation
rotation symmetry breaking

global rotation around origin = local translation + local rotation

physical excitations ⇔ broken local symmetries
local decomposition

spacetime symmetry in relativistic system = local translation + local Lorentz + local rescaling

broken by inhomogeneous condensation

nonzero spin dimensionful
local decomposition

spacetime symmetry in relativistic system = local translation + local Lorentz + local rescaling

broken by inhomogeneous condensation
nonzero spin dimensionful

ex. conformal symmetry → Poincare symmetry

dilatation ✔ ✔ ✔ ✔
special conformal ✔ ✔ ✔ ✔
local decomposition

spacetime symmetry in relativistic system = local translation + local Lorentz + local rescaling

broken by inhomogeneous condensation nonzero spin dimensionful

ex. conformal symmetry → Poincare symmetry

dilatation ✔
special conformal ✔ ✔

※ only one broken symmetry in local viewpoint ⇔ dilaton
local decomposition

spacetime symmetry in relativistic system = local translation + local Lorentz + local rescaling

local symmetry viewpoint is useful to identify excitations for broken symmetries
→ local decomposition by gauging spacetime symmetry!

ex. conformal symmetry → Poincare symmetry

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※ only one broken symmetry in local viewpoint ⇔ dilaton
gauging spacetime symmetry

spacetime symmetry in relativistic system = local translation + local Lorentz + local rescaling
gauging spacetime symmetry

spacetime symmetry in relativistic system \( \in \) diffeomorphism + local Lorentz + local Weyl

gauged by introducing curved coord & metric \( g_{\mu\nu} \) local Lorentz frame & vierbein \( e^m_\mu \)

1. Weyl gauging (always possible):
   introducing a Weyl gauge field \( W_\mu \)

2. Ricci gauging (only when conformal):
   introduce a local Weyl invariant curved spacetime action
Q. why

# of broken symmetries ≠ # of physical excitations

in spacetime symmetry case?

A. spacetime symmetry ⊊ coordinate transformation

→ local decomposition by gauging spacetime symmetry

physical excitations ⇔ broken local symmetries
plan of my talk:

1. Introduction ✔
2. Global vs local ✔
3. EFT from gauging
4. Prospects
3. EFT from gauging

- internal symmetry case
- EFT for diffs breaking
- massive modes for broken symmetries
- relation to coset construction
EFT from massive gauge bosons
unitary gauge action

# consider an internal symmetry breaking $G \rightarrow H$

$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{a} \begin{cases} \mathfrak{h} : \text{residual symmetry} \\ \mathfrak{a} : \text{broken symmetry} \end{cases}$

# unitary gauge is convenient to construct effective action

- NG modes are eaten by gauge boson
- residual H gauge symmetry

# effective action for massive gauge boson $A_\mu$:

$$\int d^4 x \ tr \left[ -\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} - \frac{v^2}{2} A_{\alpha \mu} A^\mu_\alpha + \ldots \right] \text{ with } A_{\alpha \mu} \in \mathfrak{a}$$

- $g$ : gauge coupling, $v$ : order parameter, mass = $gv$
- global symmetry limit = massless limit is singular
Stuckelberg method

# introduce NG modes by field dependent gauge transf

\[ A_\mu \rightarrow A'_\mu = \Omega^{-1} A_\mu \Omega + \Omega^{-1} \partial_\mu \Omega \]

- \[ \Omega = e^{\pi^a(x) T_a} \] with \( T_a \in \mathfrak{a} \) (broken symmetry)

- NG modes \( \pi^a = \) coordinates of \( G/H \)

NG mode sector
\[ J_\mu = \Omega^{-1} \partial_\mu \Omega \]

gauge coupling \( g \)

gauge sector
\[ A_\mu \]

# in the global symmetry limit \( g \rightarrow 0 \),

the gauge sector decouples from NG mode \( \pi \)

\[ S = \int d^4x \text{ tr} \left[ -\frac{v^2}{2} J_{a\mu} J^{a\mu} + \ldots \right] \]

\( J_{a\mu} \in \mathfrak{a} \) is the broken symmetry part of \( J_\mu = \Omega^{-1} \partial_\mu \Omega \)
1. gauge the (broken) global symmetry

2. write down the unitary gauge effective action

3. introduce NG modes by Stuckelberg method and decouple the gauge sector
unitary gauge for broken diffs

\[ \langle \phi(x) \rangle = \bar{\phi}(z) \]
unitary gauge for broken diffs

\[ \langle \phi(x) \rangle = \bar{\phi}(z) \]

- translation symmetry \( \rightarrow \langle \phi(x) \rangle = \bar{\phi}(z + c) \) is also stable
unitary gauge for broken diffs

\[ \langle \phi(x) \rangle = \bar{\phi}(z) \]

- translation symmetry \( \rightarrow \langle \phi(x) \rangle = \bar{\phi}(z + c) = \bar{\phi}(z') \)

\[ \star \text{ can be absorbed by coordinate shift } z' = z + c \]
unitary gauge for broken diffs

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- translation symmetry \( \rightarrow \langle \phi(x) \rangle = \bar{\phi}(z + c) = \bar{\phi}(z') \)

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- NG mode \( \pi : \langle \phi(x) \rangle = \bar{\phi}(z + \pi(x)) \)
unitary gauge for broken diffs

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- translation symmetry \( \rightarrow \langle \phi(x) \rangle = \bar{\phi}(z + c) = \bar{\phi}(z') \)

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※ can be absorbed by coordinate transf \( z' = z + \pi(x) \)
**unitary gauge for broken diffs**

- gauging = introduction of general coordinate system
- unitary gauge condition: $\phi(z) = \bar{\phi}(z)$
- residual 2+1 dim diffeomorphism symmetry
  - translation symmetry $\rightarrow \langle \phi(x) \rangle = \phi(z + c) = \phi(z)$
  - can be absorbed by coordinate shift $z' = z + c$
  - NG mode $\pi: \langle \phi(x) \rangle = \bar{\phi}(z + \pi(x)) = \bar{\phi}(z')$
  - can be absorbed by coordinate transff $z' = z + \pi(x)$
unitary gauge action & Stuckelberg method

- unitary gauge action (cf. EFT for inflation ['07 Cheung et al.])

\[ S = \int d^4 x \sqrt{-g} \left[ \alpha(z) + \beta(z) g^{zz}(x) + \gamma(z) (g^{zz} - 1)^2 + \ldots \right] \]

※ \( \alpha, \beta, \gamma \) depend on details of microscopic theory

- action for NG modes

1. Stuckelberg method: \( z \rightarrow z + \pi(x) \)

   e.g. \( \alpha(z) \rightarrow \alpha(z + \pi), \ g^{zz} \rightarrow g^{zz} + 2 \partial^z \pi + (\partial_\mu \pi)^2 \)

2. decouple the gauge sector ⇔ to set \( g_{\mu \nu} = \eta_{\mu \nu} \)

\[ S = \int d^4 x \left[ \alpha(z) \partial_\mu \pi \partial^\mu \pi + 4 \gamma(z) (\partial_z \pi)^2 + \mathcal{O}(\pi^3) \right] \]

※ background (bulk) eom \( \alpha(z) = \beta(z) \) is used
# second order action for diffs breaking

let us take a closer look at the simplest action

\[ S_2 = \int d^4x \, \alpha(z) \partial_\mu \pi \partial^\mu \pi \]

- free function \( \alpha(z) \) : nonzero where translation is broken

ex. 1 domain wall

outside the brane:
no kinetic term \( \rightarrow \) no NG mode
※ Nambu-Goto action in low energy limit

ex. 2 periodic modulations [cf. ’15 Hidaka-Kamikado-Kanazawa-TN]
a nonrelativistic analogue shows
dispersion is strongly anisotropic

\[ E \sim 0 \cdot k_{x,y}^2 + B k_{x,y}^4 + C k_z^2 \]

\( \rightarrow \) instability \( @ \) finite temp in large volume
massive modes associated with broken symmetries
# nonzero spin brane

**symmetry breaking**

in global sense:

translation and Lorentz invariance are broken

in local sense:

z-diffeo & z-$\mu$ local Lorentz are broken

※ introduce $g_{\mu \nu}$ and $e^{m}_{\mu}$ to gauge spacetime symmetry

→ translation modes and spin modes can be eaten

※ in such a unitary gauge,

dynamical dof: metric $g_{\mu \nu}$, vierbein $e^{m}_{\mu}$

residual symmetry: (1+2)-dim diffeo x local Lorentz
# effective action

decompose action schematically as $S = S_P + S_L + S_{PL}$

- $S_P$ : breaks the time diffs

$$S_P = \int d^4x \left[ \alpha (\partial_\mu \pi)^2 + \gamma (\partial_z \pi)^2 + \mathcal{O}(\pi^3) \right]$$

  ※ kinetic terms for $\pi$

  ※ invariant under the shift $\pi \rightarrow \pi + \text{const.}$

- $S_L$ : breaks the local Lorentz

$$S_L = -\frac{1}{2} \int d^4x \left[ \alpha_1 (\nabla_\mu e^3_\nu)^2 + \alpha_2 (\nabla_\mu e^3_\nu) (\nabla^\nu e^\mu 3) + \alpha_3 (e^\mu_3 \nabla_\mu e^3_\nu)^2 \right]$$
# effective action

decompose action schematically as $S = S_P + S_L + S_{PL}$

- $S_P$: breaks the time diffs

$$S_P = \int d^4 x \left[ \alpha (\partial_\mu \pi)^2 + \gamma (\partial_z \pi)^2 + \mathcal{O}(\pi^3) \right]$$

※ kinetic terms for $\pi$
※ invariant under the shift $\pi \rightarrow \pi + \text{const.}$

- $S_L$: breaks the local Lorentz

$$S_L = -\frac{1}{2} \int d^4 x \left[ \alpha_1 (\partial_\mu \xi_\nu)^2 + \alpha_2 (\partial_\mu \xi_\nu)(\partial^{\mu} \xi^{\nu}) + (\alpha_1 + \alpha_3)(\partial_z \xi_\nu)^2 \right]$$
# effective action

decompose action schematically as \( S = S_P + S_L + S_{PL} \)

- \( S_P \) : breaks the time diffs

\[
S_P = \int d^4x \left[ \alpha (\partial_\mu \pi)^2 + \gamma (\partial_z \pi)^2 + O(\pi^3) \right]
\]

※ kinetic terms for \( \pi \)
※ invariant under the shift \( \pi \to \pi + \text{const.} \)

- \( S_L \) : breaks the local Lorentz

\[
S_L = -\frac{1}{2} \int d^4x \left[ \alpha_1 (\partial_{\hat{\mu}} \xi_{\hat{\nu}})^2 + \alpha_2 (\partial_{\hat{\mu}} \xi_{\hat{\nu}})(\partial_{\hat{\nu}} \xi_{\hat{\mu}}) + (\alpha_1 + \alpha_3)(\partial_z \xi_{\hat{\nu}})^2 \right]
\]

※ kinetic terms for spin modes \( \xi_{\hat{\mu}} \ (\hat{\mu} = t, x, y) \)
※ invariant under the shift \( \xi_{\hat{\mu}} \to \xi_{\hat{\mu}} + \text{const.} \)
# effective action

decompose action schematically as \( S = S_P + S_L + S_{PL} \)

- \( S_{PL} \) : breaks both of diffs & local Lorentz

\[
S_{PL} = \int d^4x \sqrt{-g} m^2 (e_{3\mu} n_\mu - 1) \rightarrow \int d^4x \left[ -\frac{m^2}{2} (\xi_{\hat{\mu}} - \partial_{\hat{\mu}} \pi)^2 + \ldots \right]
\]

※ mass term for \( \xi_{\hat{\mu}} \) and mixing of \( \xi_{\hat{\mu}} \) & \( \pi \)

※ invariant under the global rotation

\[
\xi_1 \rightarrow \xi_1 + \omega_{13}, \quad \pi \rightarrow \pi - \omega_{13} x \quad (\omega_{13} : \text{const.})
\]

mass term is not prohibited by symmetry

essentially because global rotation transforms \( \pi \) also
more generally,

NG fields for local Lorentz and local Weyl become massive when they are broken at the same time as diffeomorphism
EFT from gauging

1. gauging spacetime symmetries

2. take the unitary gauge (broken symmetry modes are eaten)

3. unitary gauge action based on residual symmetry

4. Stuckelberg method & decouple the gauge sector

in contrast to the internal symmetry case,

- coefficient functions of coord when diffs are broken

- massive mode associated with broken symmetries

- local picture is important to identify such massive modes
relation to coset construction
- introduced fields $\pi^a$ for all the broken global symmetries generically include massive & unphysical ones

- $\pi^a$ have a mass term when $[P_\mu, T_a]$ contains broken generators

$$ (f^b_{aP_\mu} \neq 0) $$

- if we are interested in the massless sector only, we may impose the so-called inverse Higgs constraints

$$ a^b_\mu \sim \partial_\mu \pi^b + f^b_{aP_\mu} \pi^a + \ldots = 0 \ \text{for} \ f^b_{aP_\mu} \neq 0 $$

to remove massive and unphysical ones
coset construction
include unphysical ones
to realize broken symmetry
inverse Higgs constraints

EFT from gauging
physical ones only

integrate out massive & unphysical ones
EFT based on local symmetry viewpoint is
- more economical in particular when including massive modes
- convenient to apply gravitational systems such as cosmology

coset construction
4. Prospects
prospects1: applications to cosmology

# massive NG modes during inflation? [to appear w/Delacretaz-Senatore]
- cosmological collider physics (named by Arkani Hamed-Maldacena)
  ※ massive fields with mass $m \lesssim H$ affects inflation dynamics
  
  mass can be read off from primordial non-Gaussianity
  
  [Chen-Wang, Baumann-Green, TN-Yamaguchi-Yokoyama, Arkani Hamed-Maldacena]

- one realization via time diffs + local boost breaking

# test of isotropy during inflation [in progress w/Gong-Shiu-Soda-Yamaguchi]
- how accurate our assumption of isotropic universe?
  anisotropic background: $ds^2 = -dt^2 + a^2 e^{2\sigma} (dx^2 + dy^2) + a^2 e^{-4\sigma} dz^2$
  ※ anisotropy is characterized by $\dot{\sigma} \sim$ rotation symmetry breaking scale
  ※ effective action is constructed similarly to nonzero spin brane case
  ※ anisotropy of primordial spectrum, primordial gravitational waves, ...
other directions:

• more on nonrelativistic case
  - finite temperatures, finite densities, ...
• hydrodynamics from spacetime symmetry viewpoint
• Wess-Zumino term vs gauge anomaly
Thanks!
gauging is useful for spacetime symmetry breaking!

- global vs local picture of spacetime symmetry breaking
  - # of physical excitations ≠ # of broken global symmetries
  - physical excitations ⇐ broken local symmetries

- EFT by gauging spacetime symmetries
  - spacetime symmetry ∈ diffeo x local Lorentz x (an)isotropic Weyl
  - unitary gauge & Stuckelberg method
  - correct identification of physical excitations

- coset construction revisited
  - classification of physical meaning of inverse Higgs constraints
    ① remove redundant NG fields
    ② integrate out massive mode associated with broken symmetry