Photon Couplings to Baryon Resonances in Chiral Unitary Model

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Contents

1. *The Purpose of the Study*
2. *Introduction to Chiral Unitary Model (ChUM)*
3. *Photon Couplings to* $\Lambda(1405)$ *in ChUM*
4. *Summary and Future Work*

1. The Purpose of the Study

The Purpose of the Study

++ $\Lambda(1405)$ is a "strange" baryon ++

- $\Lambda(1405)$ --- $J^P = 1/2^-$, (uds) quarks
  with mass = (1406±4) − (50±2) i MeV.
  −> the lightest baryon with $J^P = 1/2^-$,
  although having strange quark!

- We want to know the structure of $\Lambda(1405)$!
  --- Is $\Lambda(1405)$ really exact state of (uds) quarks?
  --- Bound state of kaon and proton?

The Purpose of the Study

++ How to know the structure of Λ(1405) ++

- **Λ(1405) in ChUM.**

--- we know that Λ(1405) can be represented in ChUM.

- **Form factors will tell us the structure of hadrons.**

--- we know photon couplings to baryon and meson octets.

─> We can get the form factor of Λ(1405) in ChUM!

--- and apply to other resonances via $SU(3)_f$ symmetry.

2. Introduction to ChUM

Introduction to ChUM

++ Chiral Perturbation Theory ++

- **Massless QCD has chiral symmetry.**

--- one of the most important symmetry in QCD.

$$G \equiv \mathrm{SU}(3)_{\mathrm{L}} \otimes \mathrm{SU}(3)_{\mathrm{R}} \xrightarrow{\mathrm{S}\chi\mathrm{SB}} H \equiv \mathrm{SU}(3)_{\mathrm{V}}$$

--- chiral symmetry is spontaneously broken.

—> Re-construct effective Lagrangian with (global and local) chiral symmetry invariance.

--- in this theory, we can also re-construct the breakdown of chiral symmetry in the way as same as exact QCD.

Gasser and Leutwyler *Nucl. Phys.* B250 (1985) 465, Pich *Rep. Prog. Phys.* 58 (1995) 563.

Introduction to ChUM

++ Chiral Perturbation Theory ++

- **We can construct chiral Lagrangian as follows:**

$$\begin{aligned}\mathcal{L} &= \frac{f_\pi^2}{4}\langle D_\mu U^\dagger D^\mu U + U^\dagger\chi + \chi^\dagger U\rangle \\ &\quad + \langle \bar{B} i\gamma^\mu \nabla_\mu B\rangle - M_{\mathrm{B}}\langle \bar{B}B\rangle \\ &\quad + \frac{D}{2}\langle \bar{B}\gamma^\mu\gamma^5\{u_\mu, B\}\rangle + \frac{F}{2}\langle \bar{B}\gamma^\mu\gamma^5[u_\mu, B]\rangle\end{aligned}$$

--- at lowest order in momenta.

--- kinetic and mass terms, NG bosons-to-baryons or them-to-external fields interaction terms, and so on.

Gasser and Leutwyler *Nucl. Phys.* B250 (1985) 465, Pich *Rep. Prog. Phys.* 58 (1995) 563.

Introduction to ChUM

++ Chiral Perturbation Theory ++

- **NG boson and baryon octet fields in matrices:**

$$U(x) = u(\phi)^2 = \exp(i\sqrt{2}\,\Phi/f_\pi),$$

$$\Phi(x) \equiv \frac{\lambda^a\phi^a}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$$B(x) \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix}$$

Introduction to ChUM

++ Interactions in ChPT ++

- **We can expand ChPT Lagrangian of baryons as follows:**

$$\mathcal{L}_1^{\rm B} = \langle \bar{B} i \gamma^\mu \nabla_\mu B \rangle = \langle \bar{B} i \gamma^\mu \partial_\mu B \rangle + \frac{i}{4f_\pi^2} \langle \bar{B} \gamma^\mu [[\Phi, \partial_\mu \Phi], B] \rangle + O(\Phi^4 / f_\pi^4)$$

$$\nabla_\mu B \equiv \partial_\mu B + [\Gamma_\mu, B],$$

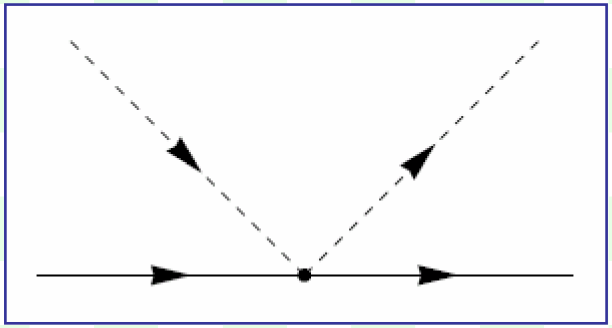
$$\Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) = \frac{1}{4f_\pi^2}[\Phi, \partial_\mu \Phi] + O(\Phi^4 / f_\pi^4)$$

--- There are kinetic term of baryons and interaction of baryons with NG bosons.

Introduction to ChUM

++ Λ(1405) in ChUM ++

- **We have a interaction term of baryons and bosons in ChPT Lagrangian:** $\frac{i}{4f_\pi^2}\langle \bar{B}\gamma^\mu[[\Phi,\partial_\mu\Phi],B]\rangle$.



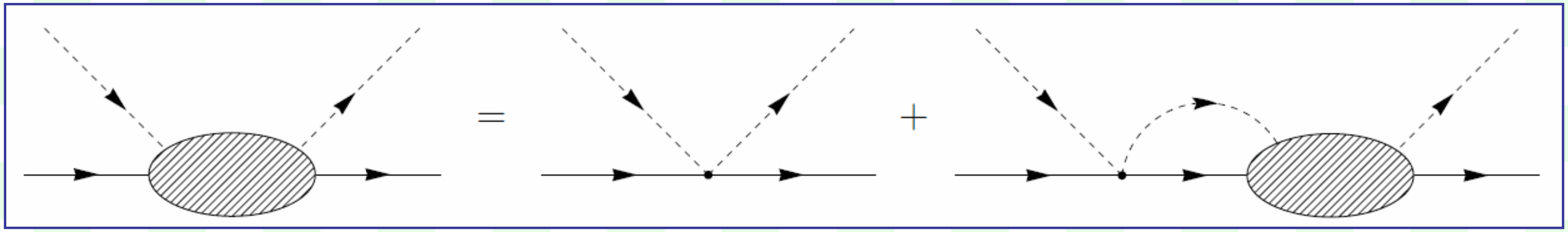
: $i\frac{C_{ji}}{4f_\pi^2}(\not{p}_1+\not{p}_3)$.

--- This should not make bound state or resonances.

―> There should be some ways to resonances!

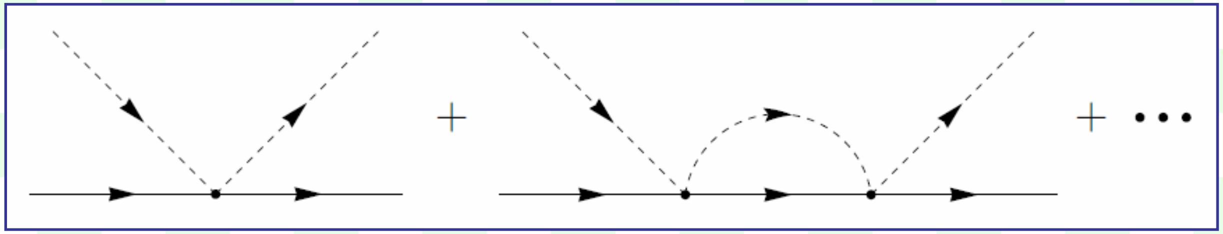
Introduction to ChUM

++ Λ(1405) in ChUM ++



- We want to solve Lippmann-Schwinger equation.

--- The meaning of this scattering diagram is following:



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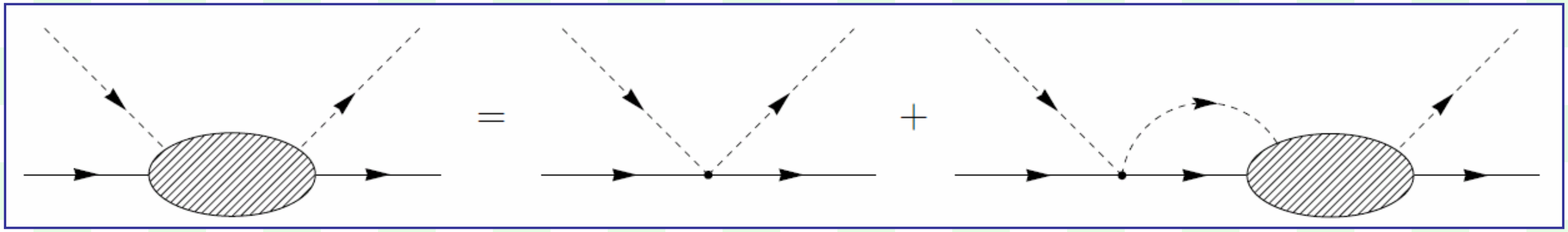
Kaiser, Siegel and Weise, *Nucl. Phys.* A594 (1995) 325,

Oset and Ramos, *Nucl. Phys.* A635 (1998) 99.

Introduction to ChUM

++ Λ(1405) in ChUM ++



- <u>Non-relativistic approximation</u> to baryons:

$$i\frac{C_{ji}}{4f_\pi^2}(\not{p}_1 + \not{p}_3) \rightarrow i\frac{C_{ji}}{4f_\pi^2}(2\sqrt{s} - M_j - M_i),$$

$$\frac{i}{\not{p} - M_i} \rightarrow \frac{2iM_i}{p^2 - M_i^2}$$

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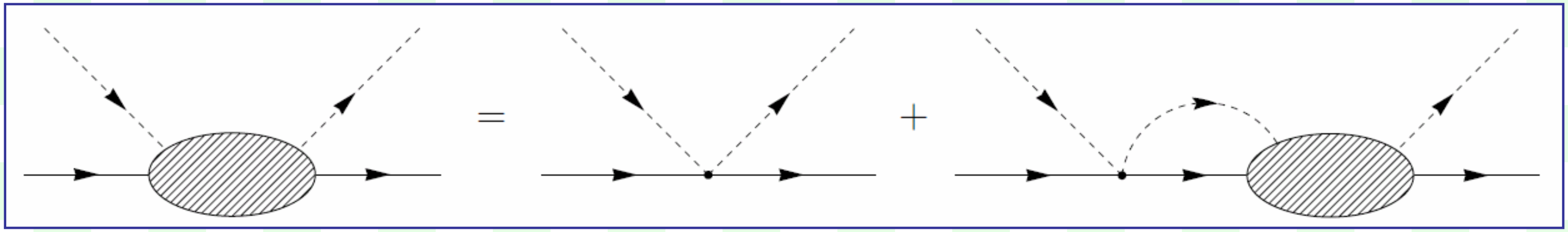
Introduction to ChUM

++ Λ(1405) in ChUM ++



- **Now we construct matrix equation:**

$$T_{ji} = V_{ji} + V_{jk} G_k T_{ki}, \quad T = (1 - VG)^{-1} V,$$

$$V_{ji} = -\frac{C_{ji}}{4 f_\pi^2} (2\sqrt{s} - M_j - M_i) \left(\frac{M_i + E}{2M_i} \right)^{\frac{1}{2}} \left(\frac{M_j + E'}{2M_j} \right)^{\frac{1}{2}},$$

$$G_k = 2 i M_k \int \frac{d^4 q_1}{(2\pi)^4} \frac{1}{q_1^2 - m_k^2} \frac{1}{(P - q_1)^2 - M_k^2}$$

--- We can solve it!

Introduction to ChUM

++ Λ(1405) in ChUM ++

- **The loop integral has a divergence.**

—> This divergence is renormalized and bring parameters:

$$G_k = 2iM_k \int \frac{d^4 q_1}{(2\pi)^4} \frac{1}{q_1^2 - m_k^2} \frac{1}{(P-q_1)^2 - M_k^2}$$
$$= \frac{2M_k}{16\pi^2}\left(a_k(\mu) + \ln\left(\frac{M_k^2}{\mu^2}\right) + \frac{m_k^2 - M_k^2 + s}{2s} \ln\left(\frac{m_k^2}{M_k^2}\right) + \frac{q_k}{\sqrt{s}} \left(\ln(s - (M_k^2 - m_k^2) + 2q_k\sqrt{s}) + \ln(s + (M_k^2 - m_k^2) + 2q_k\sqrt{s}) - \ln(-s + (M_k^2 - m_k^2) + 2q_k\sqrt{s}) - \ln(-s - (M_k^2 - m_k^2) + 2q_k\sqrt{s}) \right) \right),$$
$$q_k = \frac{(s - M_k^2 + m_k^2)^2}{4s} - m_k^2$$

Oset, Ramos and Bennhold, *Phys. Lett.* B527 (2002) 99.

Introduction to ChUM

++ Λ(1405) in ChUM ++

$$G_k = 2iM_k \int \frac{d^4 q_1}{(2\pi)^4} \frac{1}{q_1^2 - m_k^2} \frac{1}{(P-q_1)^2 - M_k^2}$$

$$= \frac{2M_k}{16\pi^2} \Bigg(a_k(\mu) + \ln\left(\frac{M_k^2}{\mu^2}\right) + \frac{m_k^2 - M_k^2 + s}{2s} \ln\left(\frac{m_k^2}{M_k^2}\right) +$$

$$\frac{q_k}{\sqrt{s}} \Big(\ln(s - (M_k^2 - m_k^2) + 2q_k\sqrt{s}) + \ln(s + (M_k^2 - m_k^2) + 2q_k\sqrt{s})$$

$$- \ln(-s + (M_k^2 - m_k^2) + 2q_k\sqrt{s}) - \ln(-s - (M_k^2 - m_k^2) + 2q_k\sqrt{s}) \Big) \Bigg),$$

$$q_k = \frac{(s - M_k^2 + m_k^2)^2}{4s} - m_k^2$$

- **These parameters are determined to have same values of cut-off formula at KN threshold.**

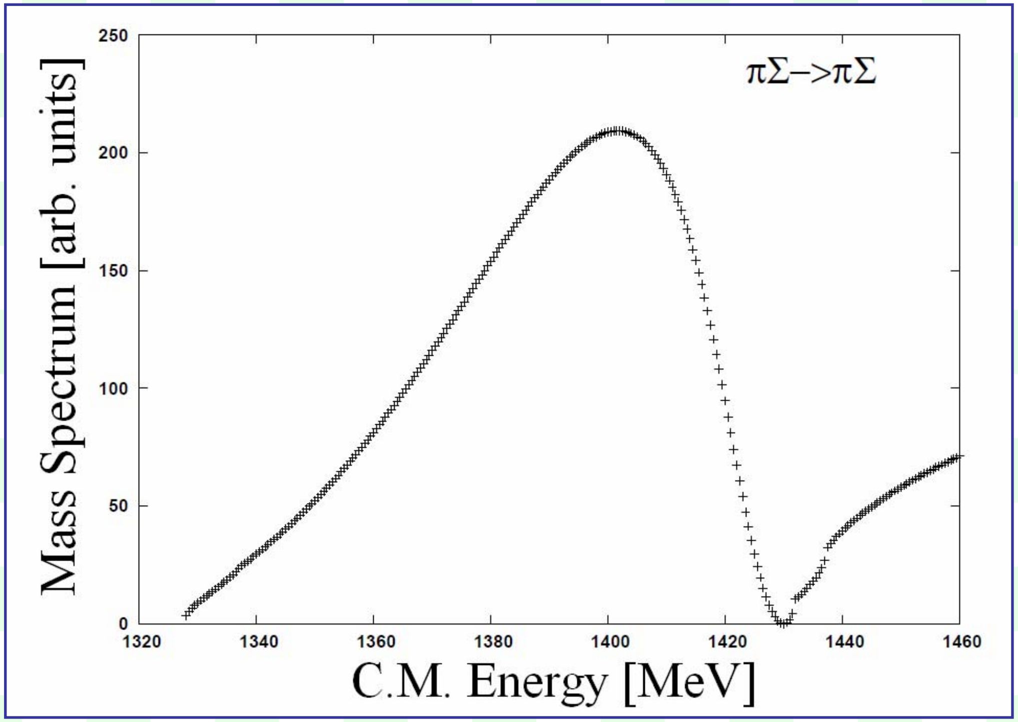
$$a_{\bar{K}N} = -1.84,\ a_{\pi\Sigma} = -2.00,\ a_{\pi\Lambda} = -1.83,$$
$$a_{\eta\Lambda} = -2.25,\ a_{\eta\Sigma} = -2.38,\ a_{K\Xi} = -2.67$$

Introduction to ChUM

++ Λ(1405) in ChUM ++

- **Mass spectrum is calculated and plotted:**

$$\frac{d\sigma}{dm} = (\text{const.})|T_{\pi\Sigma\to\pi\Sigma}|^2 p_{cm}$$



--- Physical masses are used here.

--- Agreement with data in experimental!

3. Photon Couplings to Λ(1405) in ChUM

Photon Couplings to $\Lambda(1405)$ in ChUM

++ Photon couplings to NG boson octet ++

- We can extract photon couplings to NG bosons from covariant derivative in ChPT:

$$l_\mu = r_\mu = eQA_\mu, \; Q = \frac{1}{3}\begin{pmatrix} 2 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

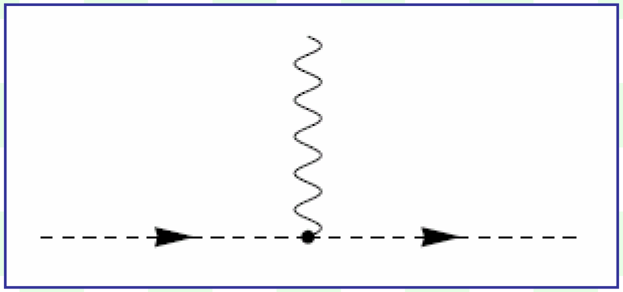
$$\begin{aligned} D_\mu U &\equiv \partial_\mu U - i r_\mu U + i U l_\mu, \\ D_\mu U^\dagger &\equiv \partial_\mu U + i U^\dagger r_\mu - i l_\mu U^\dagger \end{aligned} .$$

Photon Couplings to Λ(1405) in ChUM

++ Photon couplings to NG boson octet ++

- **We now get ChPT Lagrangian with photon couplings to NG bosons:**

$$\mathcal{L}_2 = \frac{f_\pi^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle = \frac{1}{2}\langle \partial_\mu \Phi \partial^\mu \Phi\rangle - ieA_\mu\langle \partial^\mu \Phi [Q, \Phi]\rangle + O(\Phi^4/f_\pi^2)$$



: $ieQ_{M_i}(p + p')^\mu \delta_{ij}$.

Photon Couplings to $\Lambda(1405)$ in ChUM

++ Photon couplings to baryon octet ++

- We can extract photon couplings to baryons from covariant derivative in ChPT:

$$l_\mu = r_\mu = eQA_\mu,\ Q = \frac{1}{3}\begin{pmatrix} 2 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

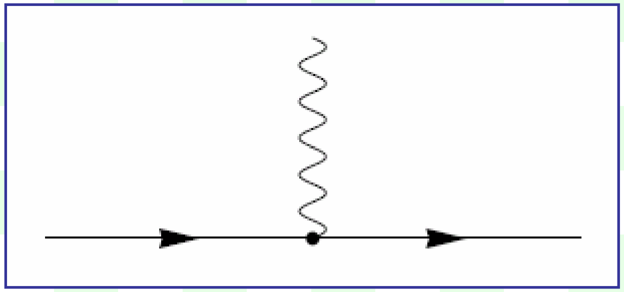
$$\Gamma_\mu = \frac{1}{2}\left(u^\dagger(\partial_\mu - ieQA_\mu)u + u(\partial_\mu - ieQA_\mu)u^\dagger\right) = -iQA_\mu + O(\Phi^2/f_\pi^2)$$

Photon Couplings to Λ(1405) in ChUM

++ Photon couplings to baryon octet ++

- **We now get ChPT Lagrangian with photon couplings to baryons:**

$$\mathcal{L}_1^{\mathrm{B}} = \langle \bar{B} i \gamma^\mu \nabla_\mu B \rangle = \langle \bar{B} i \gamma^\mu \partial_\mu B \rangle + e A_\mu \langle \bar{B} \gamma^\mu [Q, B] \rangle + O(\Phi^2 / f_\pi^2)$$



: $ieQ_{B_i} \gamma^\mu \delta_{ij}$.

Photon Couplings to $\Lambda(1405)$ in ChUM

++ Ward identity ++

- For calculating form factors, We have to make photon couple to baryon resonances, such as $\Lambda(1405)$, within **gauge invariance**.

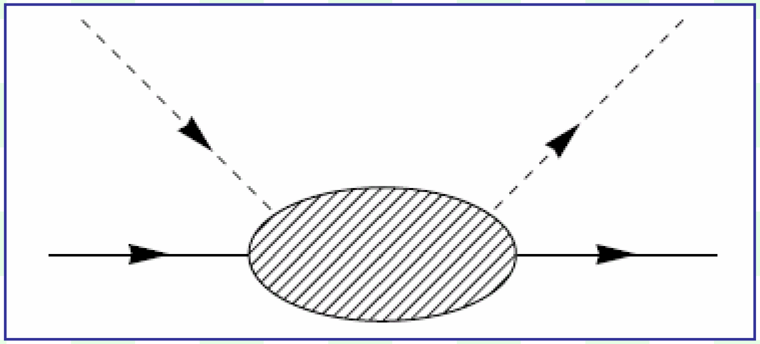
—> Photons couple to everywhere they can couple.

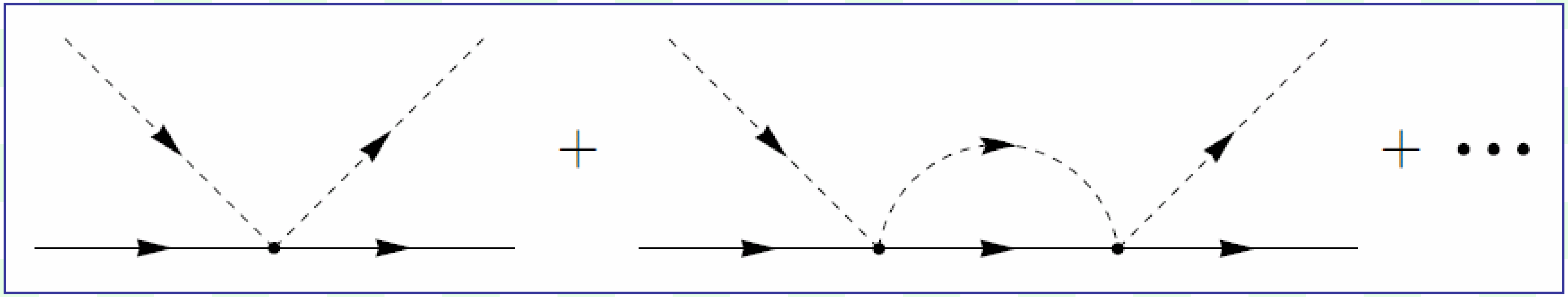
- We can use **"Ward identity"** to confirm whether we get correct photon couplings: $q_\mu T^\mu = 0$

Photon Couplings to $\Lambda(1405)$ in ChUM

++ Ward identity ++

- **Regarding the resonances as interactions of baryon and meson as below, we can understand where photon couples.**



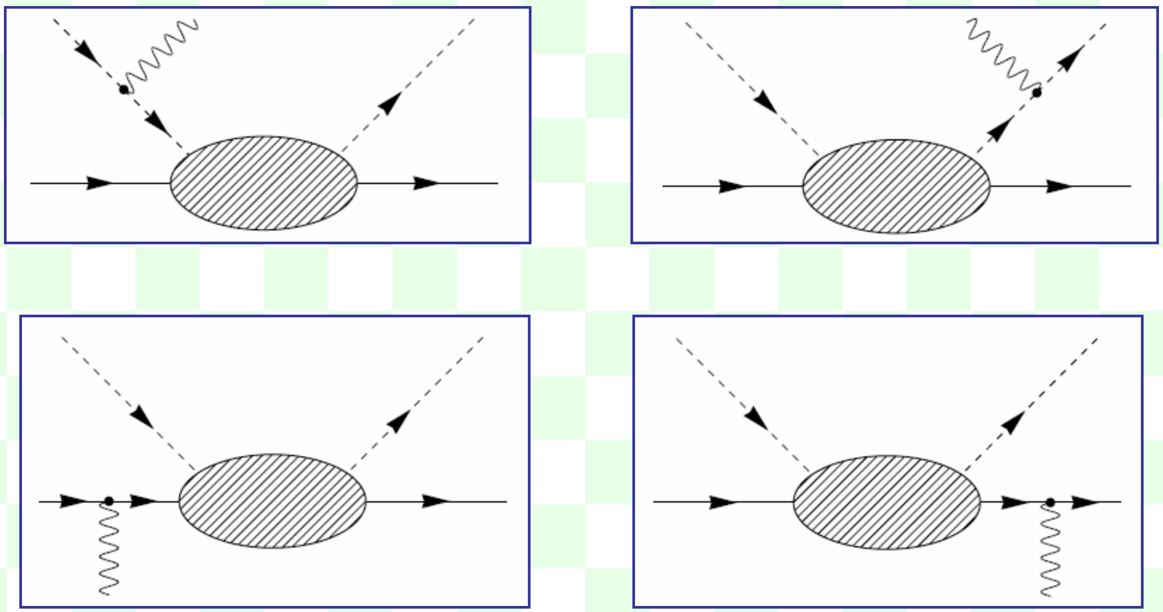


--- Photon couples to propagating lines and vertices.

Photon Couplings to Λ(1405) in ChUM

++ Ward identity ++

- We have **10 diagrams** with photon couplings(1):

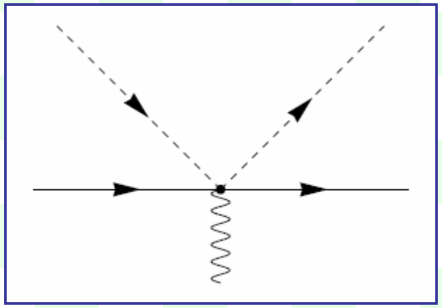


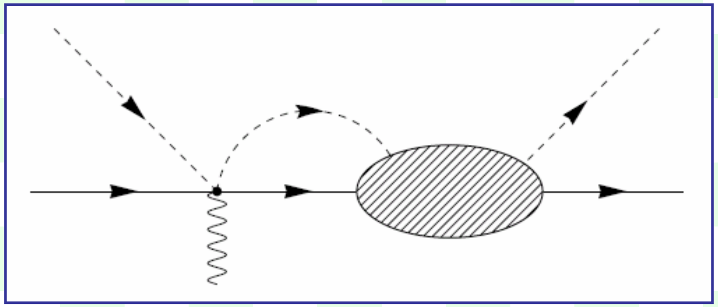
Borasoy, Bruns, Meiβner and Niβler, *Phys. Rev.* C72 (2005) 065201.

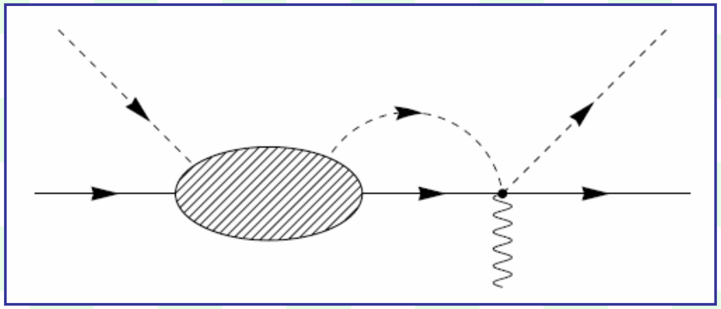
Photon Couplings to Λ(1405) in ChUM

++ Ward identity ++

- We have **10 diagrams** with photon couplings(2):





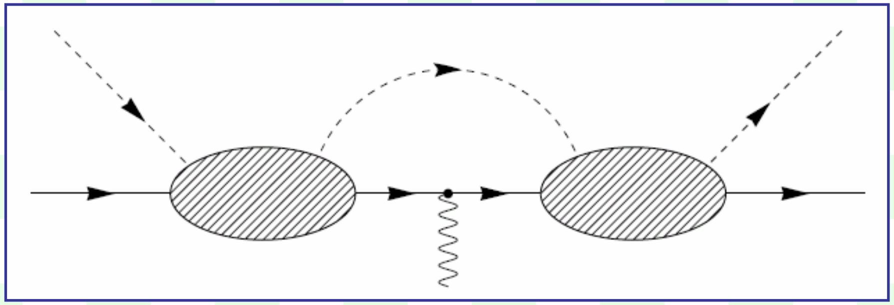


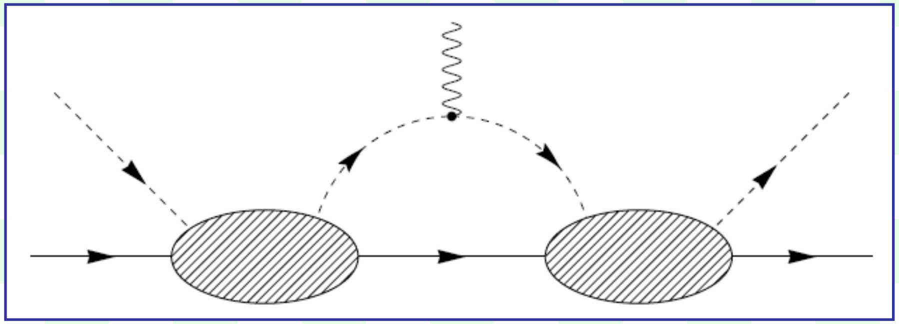
Borasoy, Bruns, Meiβner and Niβler, *Phys. Rev.* C72 (2005) 065201.

Photon Couplings to Λ(1405) in ChUM

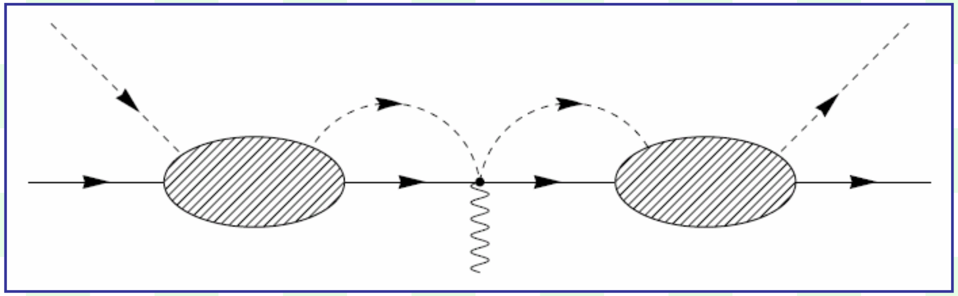
++ Ward identity ++

- We have **10 diagrams** with photon couplings(3):





- We can confirm Ward identity with these 10 diagrams.



Borasoy, Bruns, Meiβner and Niβler, *Phys. Rev.* C72 (2005) 065201.

4. Summary and Future Work

Summary and Future Work

++ Summary ++

- We want to calculate form factors of $\Lambda(1405)$ and other baryon resonances in ChUM.

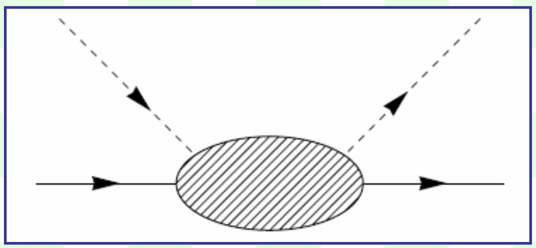
- We studied photon couplings to $\Lambda(1405)$ with gauge invariance.

—> We need **10 diagrams** in all.

Summary and Future Work

++ Future work ++

- **Calculations of form factors:**



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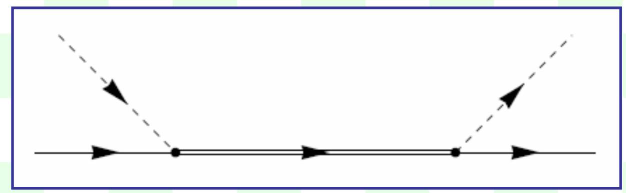


$$-iT_{ji}$$

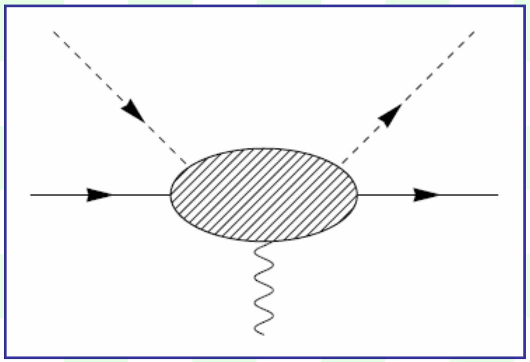
–>

$$ig_j \frac{i}{\not{P} - M_\Lambda} ig_i$$

Summary and Future Work

++ Future work ++

- **Calculations of form factors:**



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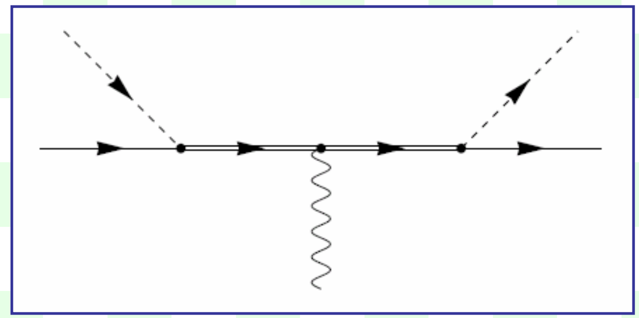

$$-iT^{\mu}_{ji} \quad \text{–>} \quad ig_j \frac{i}{\not{P} + \not{q} - M_\Lambda} (\mathrm{F.F.})^\mu \frac{i}{\not{P} - M_\Lambda} ig_i$$

Summary and Future Work

++ Future work ++

- **Calculations of form factors:**

$$-iT_{ji} \to ig_j \frac{i}{\not{P} - M_\Lambda} ig_i$$

$$-iT^\mu_{ji} \to ig_j \frac{i}{\not{P} + \not{q} - M_\Lambda} (\mathrm{F.F.})^\mu \frac{i}{\not{P} - M_\Lambda} ig_i$$

—> $$(\mathrm{F.F.})^\mu = \lim_{z \to z_\Lambda} (z - z_\Lambda) \frac{-iT^\mu_{ji}(z)}{-iT_{ji}(z)}$$

--- removing remaining pole in propagator.

Summary and Future Work

++ Future work ++

- Approximation within gauge invariance.
- Radius of $\Lambda(1405)$ from the form factor.
- Magnetic form factors.
- Form factors of baryon and NG boson octets.
- Applications to other resonances.

To be continued…