T-duality, Fiber Bundles and Matrices

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との共同研究に基づく

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String Theory

「ものの最小にして究極の構成単位は
　ひも状の物質である、と考える物理理論」

閉弦…重力
開弦…ゲージ相互作用

相互作用の統一理論

摂動論だけでは不完全 ⇒ 非摂動効果
・D-brane
・AdS/CFT対応 (ゲージ/重力対応)

超弦理論は重力を含む理論なので時空も動的に決まるはず

行列模型

Matrix models

ゲージ理論を実現するreduced model [1980's]

・Eguchi-Kawai 模型 [Eguchi-Kawai]

超弦理論における行列模型

・IIB matrix model [Ishibashi-Kawai-Kitazawa-Tsuchiya]

・Matrix theory [Banks-Fischler-Shenker-Susskind]

☆曲がった時空を行列で実現

IIB行列模型で曲がった時空を記述 [Hanada-Kawai-Kimura]

★行列の自由度から時空が出てくるメカニズムの一つ

ゲージ理論におけるT-duality [Taylor]

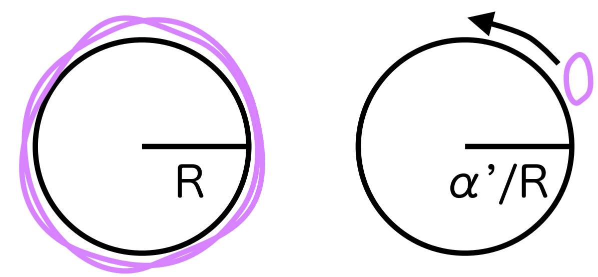
曲がった時空上のゲージ理論と行列の関係

Contents

T-duality

T-duality

巻きつき数 ⇔ 運動量、半径R ⇔ α'/R
で理論が不変



・開弦の解放端
・Dp-brane [p+1次元]

⇔

・開弦の固定端
・D(p-1)-brane [p次元]

★T-duality for gauge theory [Taylor]

Dp-brane上のU(N)Yang-Mills

↓ dimensional reduction　　↕ T-duality

D(p-1)-brane上のU(N×∞)Yang-Mills+Higgs

Dimensional reduction

Dp-brane : U(N)YM on $R^p \times S^1$

$$z^M = (x^\mu, y)$$
$$y \sim y + 2\pi R$$

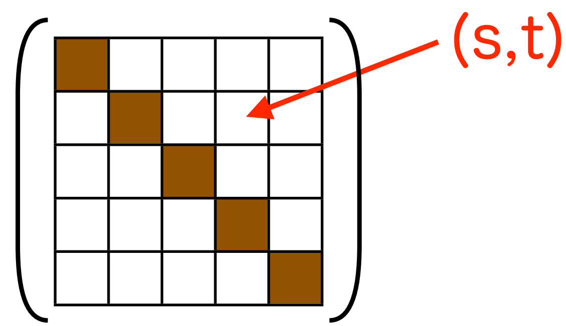
$$S_{p+1} = \frac{1}{g_{p+1}^2} \int d^{p+1}z \, \frac{1}{4} \mathrm{Tr}(F_{MN}F_{MN})$$

dimensional reduction

$$A_\mu \to a_\mu(x)$$
$$A_y \to \phi(x)$$

D(p-1)-brane : U(N)YM + Higgs on R^p

$$S_p = \frac{1}{g_p^2} \int d^p x \, \mathrm{Tr} \left(\frac{1}{4} f_{\mu\nu} f_{\mu\nu} + \frac{1}{2} D_\mu \phi D_\mu \phi \right)$$



U(N×∞)を考える：N×Nブロックを並べる

$$S_p = \frac{1}{g_p^2} \int d^p x \sum_{s,t} \mathrm{tr} \left(\frac{1}{4} f_{\mu\nu}^{(s,t)} f_{\mu\nu}^{(t,s)} + \frac{1}{2} (D_\mu \phi)^{(s,t)} (D_\mu \phi)^{(t,s)} \right)$$

Taylor's recipe

Taylor's recipe

$$U a_\mu U^\dagger = a_\mu$$
$$U \phi U^\dagger = \phi + 2\pi \tilde{R} \mathbf{1}_{N\times\infty}$$

$$U = \begin{pmatrix} \mathbf{0}_N & \mathbf{1}_N & & & \\ & \mathbf{0}_N & \mathbf{1}_N & & \\ & & \mathbf{0}_N & \mathbf{1}_N & \\ & & & \mathbf{0}_N & \mathbf{1}_N \\ & & & & \mathbf{0}_N \end{pmatrix}$$

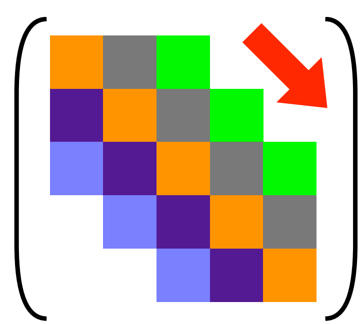
background + fluctuation : $a_\mu = \hat{a}_\mu + \tilde{a}_\mu$

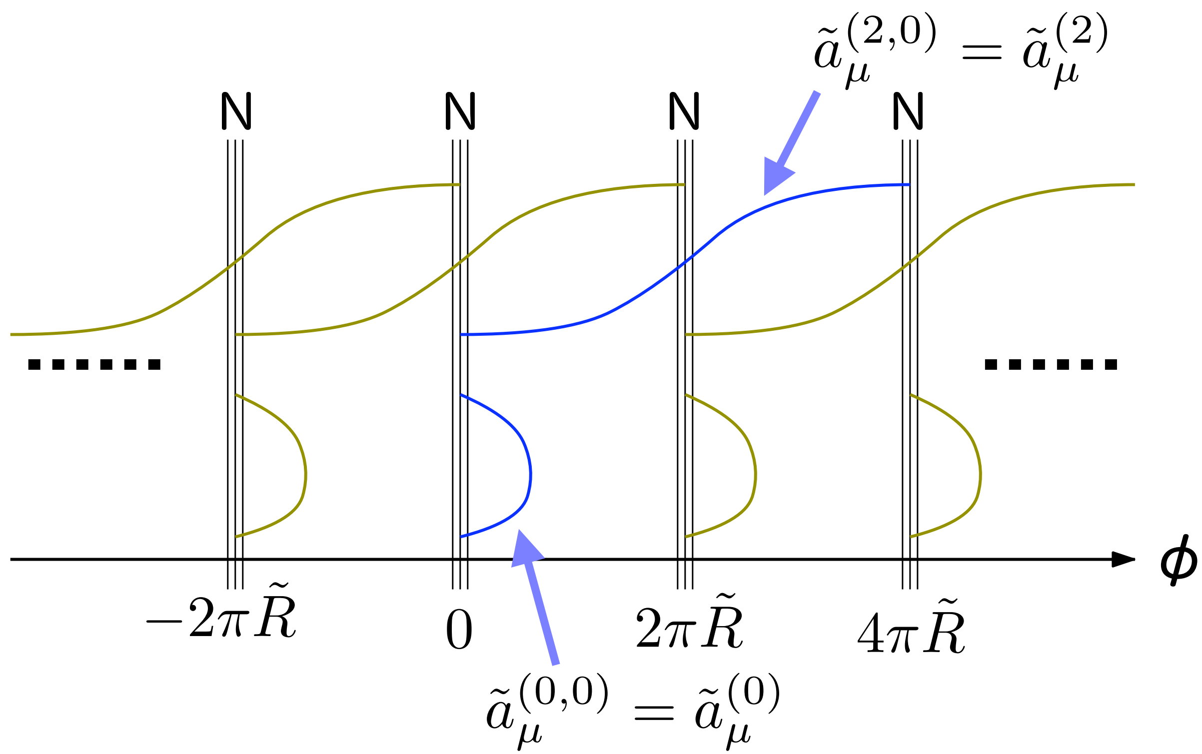
background … 対角行列

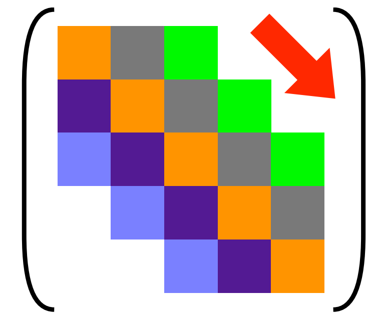
$$\hat{a}_\mu = 0$$
$$\hat{\phi} = 2\pi \tilde{R}\,\mathrm{diag}(\cdots, \underbrace{s-1, \cdots, s-1}_{N}, \underbrace{s, \cdots, s}_{N}, \underbrace{s+1, \cdots, s+1}_{N}, \cdots)$$

fluctuation … orbifolding条件

$$\tilde{a}_\mu^{(s+1,t+1)} = \tilde{a}_\mu^{(s,t)} \equiv \tilde{a}_\mu^{(s-t)}$$
$$\tilde{\phi}^{(s+1,t+1)} = \tilde{\phi}^{(s,t)} \equiv \tilde{\phi}^{(s-t)}$$







D-braneを並べて周期性を課す ⇒ S^1 : (s-t)は巻き付き数

T-dual : winding number (s-t) ⇔ momentum w

D(p-1)-braneの場でDp-braneの場を定義

$$A_\mu(x,y) \equiv \sum_w \tilde{a}_\mu^{(w)}(x) e^{-\frac{i}{R}wy}$$

$$A_y(x,y) \equiv \sum_w \tilde{\phi}^{(w)}(x) e^{-\frac{i}{R}wy}$$

$$R = \frac{1}{2\pi\tilde{R}}$$

$$(D_\mu \phi(x))^{(s,t)} = \partial_\mu \tilde{\phi}^{(s-t)}(x) + i2\pi\tilde{R}(s-t)\tilde{a}_\mu^{(s-t)}(x) \\ -i\sum_u (\tilde{a}_\mu^{(s-u)}(x)\tilde{\phi}^{(u-t)}(x) - \tilde{\phi}^{(s-u)}(x)\tilde{a}_\mu^{(u-t)}(x))$$

$$= \frac{1}{2\pi R}\int_0^{2\pi R} dy(\partial_\mu A_y(x,y) - \partial_y A_\mu(x,y) - i[A_\mu(x,y), A_y(x,y)])e^{\frac{i}{R}(s-t)y}$$

$$= \frac{1}{2\pi R}\int_0^{2\pi R} dy F_{\mu y}(x,y)e^{\frac{i}{R}(s-t)y}$$

$$f_{\mu\nu}^{(s,t)}(x) = \frac{1}{2\pi R}\int_0^{2\pi R} dy F_{\mu\nu}(x,y)e^{\frac{i}{R}(s-t)y}$$

$$S_p = \frac{1}{g_p^2}\int d^p x \sum_{s,t} \mathrm{tr}\left(\frac{1}{4}f_{\mu\nu}^{(s,t)}f_{\mu\nu}^{(t,s)} + \frac{1}{2}(D_\mu\phi)^{(s,t)}(D_\mu\phi)^{(t,s)}\right)$$

$$= \frac{1}{g_p^2}\frac{1}{2\pi R}\sum_w \int d^{p+1}z\, \frac{1}{4}\mathrm{tr}(F_{MN}F_{MN})$$

S_{p+1}に帰着

Contents

Consistent truncation

pure-gauge background $V = \mathrm{diag}(\cdots, \underbrace{e^{\frac{i}{R}n_s y}, \cdots, e^{\frac{i}{R}n_s y}}_{N_s}, \cdots)$

$A_M = 0$ ⟷ (gauge transf.) $\hat{A}_M = -i\partial_M V V^\dagger$

$y \sim y + 2\pi R, \ n_s \in \mathbb{Z}$

$A^{(s,t)}_{M,0}$ ↓ $\hat{a}_\mu = 0$, $\hat{\phi} = 0$

$A^{(s,t)}_{M,-(n_s-n_t)}$, $\tilde{A}^{(s,t)}_{M,0}$ ↓

$\hat{a}_\mu = 0$ ⟷ (gauge transf.) ✕

$\hat{\phi} = \frac{1}{R}\mathrm{diag}(\cdots, \underbrace{n_s, \cdots, n_s}_{N_s}, \cdots)$

mode expansion

$$A^{(s,t)}_M = \sum_m A^{(s,t)}_{M,m} e^{\frac{i}{R}my}$$

$$\tilde{A}^{(s,t)}_M = \sum_m \tilde{A}^{(s,t)}_{M,m} e^{\frac{i}{R}my}$$

⇒ relation between modes

$$A^{(s,t)}_{M,m-(n_s-n_t)} = \tilde{A}^{(s,t)}_{M,m}$$

T-dualityは Ns=N, ns=s, (s,t)ブロックは(s-t)のみに依る
のとき成立

Contents

Principal U(1) bundle case

Taylorの方法を曲がった空間上のゲージ理論に拡張する
特に主U(1)ファイバー束の場合を考える

座標系(patch)を張り合わせる必要がある

patch同士はゲージ変換で移りあう

各patchでファイバー方向を見出す

各patchでFourier変換

Dimensional reduction

Local metric on the total space ([I]はpatchのラベル)

$$ds^2_{D+1} = g^{[I]}_{\mu\nu} dx^\mu_{[I]} dx^\nu_{[I]} + (dy_{[I]} + b^{[I]}_\mu dx^\mu_{[I]})^2$$

U(N) YM on the total space

$$S_{D+1} = \frac{1}{g^2_{D+1}} \int d^{D+1}z \, \sqrt{G} \frac{1}{4} \mathrm{Tr}(F_{AB} F_{AB})$$

↓ dimensional reduction

$$A_\alpha = a_\alpha$$
$$A_{D+1} = \phi$$

U(N) YM + Higgs on the base space

$$S_D = \frac{1}{g^2_D} \int d^D x \sqrt{g} \, \mathrm{Tr} \left(\frac{1}{4} (f_{\alpha\beta} + b_{\alpha\beta}\phi)(f_{\alpha\beta} + b_{\alpha\beta}\phi) + \frac{1}{2} D_\alpha \phi D_\alpha \phi \right)$$

pure-gauge background

$$A_A = 0$$

gauge transf. ⟷

$$\hat{A}^{[I]}_{\alpha} = -\frac{1}{R} b^{[I]}_{\alpha}\, \mathrm{diag}(\cdots, \underbrace{n_s, \cdots, n_s}_{N_s}, \cdots)$$

$$\hat{A}_{D+1} = \frac{1}{R}\mathrm{diag}(\cdots, \underbrace{n_s, \cdots, n_s}_{N_s}, \cdots)$$

U(N)のコピー

$A^{[I](s,t)}_{A,-(n_s-n_t)}$ ↘ $\tilde{A}^{[I](s,t)}_{A,0}$ ↓

$$\hat{a}^{[I]}_{\alpha} = -b^{[I]}_{\alpha}\hat{\phi}$$ ← monopole-like configuration

$$\hat{\phi} = 2\pi\tilde{R}\, \mathrm{diag}(\cdots, \underbrace{n_{s-1}, \cdots, n_{s-1}}_{N_{s-1}}, \underbrace{n_s, \cdots, n_s}_{N_s}, \underbrace{n_{s+1}, \cdots, n_{s+1}}_{N_{s+1}}, \cdots)$$

U(N×∞)

T-dualityはN_s=N, n_s=s, (s,t)ブロックは(s-t)のみに依るのとき成立

T-dual : monopole charge ⇔ momentum

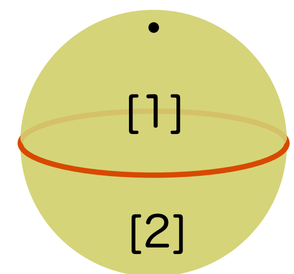
Examples of U(1) bundle

- $S^3 \to S^2$

$$ds^2_{S^3} = d\theta^2 + \sin^2\theta d\phi^2 + (d\psi + \cos\theta d\varphi)^2$$

$$\begin{cases} y_{[1]} = \psi + \varphi \\ y_{[2]} = \psi - \varphi \end{cases}$$

$\theta = 0$



backgrounds on S^2

$$\hat{a}_1 = 0$$

$$\hat{a}_2^{[1]} = \tan\frac{\theta}{2}\,\hat{\phi}, \quad \hat{a}_2^{[2]} = -\cot\frac{\theta}{2}\,\hat{\phi}$$ ← Dirac monopole

$$\hat{\phi} = \frac{1}{2}\mathrm{diag}(\cdots, \underbrace{s-1,\cdots,s-1}_{N}, \underbrace{s,\cdots,s}_{N}, \underbrace{s+1,\cdots,s+1}_{N}, \cdots)$$

- $S^5 \to CP^2$
- Heisenberg nilmanifold $\to T^2$

Curved D-brane ?

曲がった空間に巻き付いているDp-brane

$$G_{MN}dz^M dz^N = g_{\mu\nu}dx^\mu dx^\nu + (dy + b_\mu dx^\mu)^2$$

$$B_{\mu\nu} = 0$$

↕ T-duality (Buscher's rule)

D(p-1)-brane

$$G'_{MN}dz^M dz^N = g_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$B'_{\mu\nu} = B'_{yy} = 0 \quad B'_{y\mu} = b_\mu$$

DBI action

$$S_p = \tau_p \int dx^p \sqrt{\det(\tilde{G}_{\mu\nu} + \tilde{B}_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}$$

$$\sim \frac{1}{g_p^2} \int dx^p \sqrt{g} \left(\frac{1}{4}(f_{\alpha\beta} + b_{\alpha\beta}\phi)^2 + \frac{1}{2}(\partial_\alpha \phi)^2 \right)$$

Summary and Outlook

Summary

・TaylorによるT-dualityの行列での解釈を
　非自明なU(1)ファイバー束の場合に拡張した

Outlook

・ファイバーが一般の群の場合
・(色々な)曲がった空間上のゲージ理論を行列模型で実現
・曲がった時空上の超弦理論を行列模型で実現