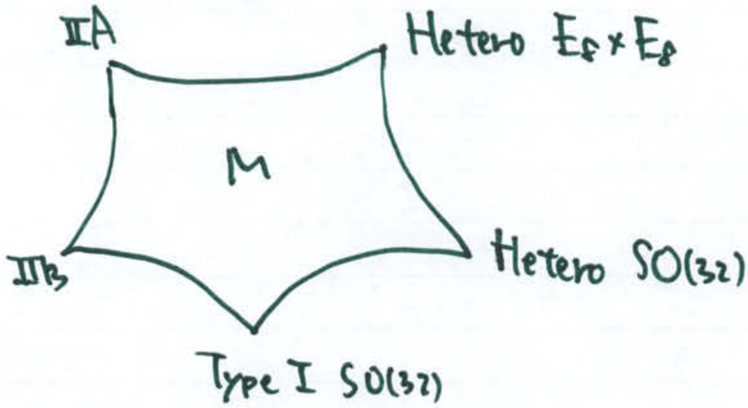
超弦理論におけるD-braneと双対性について

阪大　百武慶文



§ Bosonic String

§ Superstring

§ T-duality

§ S-duality

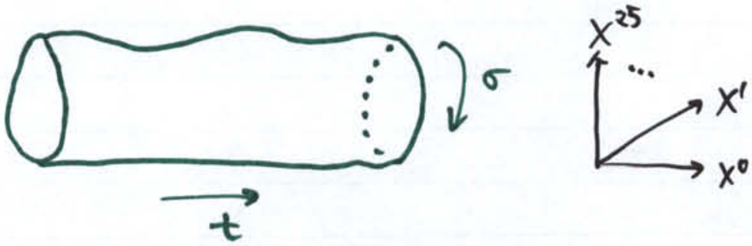
§ Non Abelian Gauge Symmetry

関西地域セミナー

Bosonic String

- String の作用

平坦な時空を伝播する closed string



作用は string がはく面積で与えられる

$$S_{NG} = -T \times \text{面積} \qquad \text{Nambu-Goto 作用}$$

$$\left(\begin{array}{l} g_{\alpha\beta} = \dfrac{\partial X^\mu}{\partial \xi^\alpha} \dfrac{\partial X^\nu}{\partial \xi^\beta} \eta_{\mu\nu} \\ \text{面積} = \int d^2\xi \sqrt{-\det g_{\alpha\beta}} \end{array} \right.$$

量子化には Polyakov 作用が必要

$$S[g, X] = -\frac{T}{2} \int d^2\xi \sqrt{-g}\, g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

(注) NG 作用と Polyakov 作用は古典的には等価

$$S_{NG} = S[g_{solution}, X]$$

$$\left(\left. \frac{\delta S[g, X]}{\delta g} \right|_{g = g_{solution}} = 0 \right.$$

- Polyakov 作用の量子化

まず対称性を利用して作用を簡単にする

1. Diffeo $\quad g'_{\alpha\beta}(\xi') = \dfrac{\partial \xi^\rho}{\partial \xi'^\alpha} \dfrac{\partial \xi^\sigma}{\partial \xi'^\beta} g_{\rho\sigma}(\xi)$ $\quad$ 2

2. Weyl $\quad g'_{\alpha\beta}(\xi) = \Omega(\xi)\, g_{\alpha\beta}(\xi)$ $\quad$ 1

この2つを使って計量を平坦にできる

$$g_{\alpha\beta} = \eta_{\alpha\beta} \qquad \text{covariant gauge}$$

作用と拘束条件は簡単になる

- $$S[X] = -\frac{T}{2}\int d^2\xi \left(-\dot{X}^\mu \dot{X}_\mu + X'^\mu X'_\mu\right)$$
  $$= -T\int d\xi^+ d\xi^- \, \partial_+ X^\mu \partial_- X_\mu \,, \qquad \xi^\pm = t \pm \sigma$$

- $T_{++} = T_{--} = 0$ $\quad \left(\begin{array}{l} T_{++} = T\,\partial_+ X^\mu \partial_+ X_\mu \\ T_{--} = T\,\partial_- X^\mu \partial_- X_\mu \end{array} \right.$

- 境界条件 $\quad X^\mu(t, \sigma + 2\pi) = X^\mu(t, \sigma)$ $\quad$ closed string

さらに共形対称性を利用して Light-Cone ゲージをとる

$\xi^{+\prime} = \xi^{+\prime}(\xi^+)\,,\ \xi^{-\prime} = \xi^{-\prime}(\xi^-)$

$$X^+ = X^0 + X^{25} = x^+ + \ell_s^2 p^+ t$$

$X^- = X^0 - X^{25}$ は $T_{++} = T_{--} = 0$ を解いて決める

結局、以下を量子化することになった

- $S[X] = +T\int d\xi^+ d\xi^- \, \partial_+ X^i \partial_- X_i \;, \quad i=1,\cdots,24$

$\searrow \; \partial_+ \partial_- X^i = 0$

- $\begin{cases} \frac{1}{2} l_s^2 p^+ \partial_+ X^- = \partial_+ X^i \partial_+ X_i \\ \frac{1}{2} l_s^2 p^+ \partial_- X^- = \partial_- X^i \partial_- X_i \end{cases}$

- $X^i(t, \sigma+2\pi) = X^i(t,\sigma)$

量子化すると

$$X^i(t,\sigma) = x^i + l_s^2 p^i t + \frac{i l_s}{\sqrt{2}} \sum_{n\neq 0} \frac{1}{n}\left(\alpha_n^i e^{-in(t+\sigma)} + \tilde{\alpha}_n^i e^{-in(t-\sigma)} \right)$$

$$\begin{cases} [p^i, x^j] = -i\delta^{ij} \\ [\alpha_m^i, \alpha_{-n}^j] = m\,\delta^{i,j}\delta_{m,n} \\ [\tilde{\alpha}_m^i, \tilde{\alpha}_{-n}^j] = m\,\delta^{i,j}\delta_{m,n} \end{cases}$$

拘束条件は

$$M^2 = p^+p^- - p^i p_i = \frac{2}{l_s^2}\sum_{m=1}^{\infty}\left(\alpha_{-m}^i \alpha_m^i + \tilde{\alpha}_{-m}^i \tilde{\alpha}_m^i \right) - \frac{4}{l_s^2} \qquad \text{mass}$$

$$\frac{1}{2}\sum_{m=1}^{\infty} :\alpha_m^i \alpha^i_{-m}: = \frac{1}{2}\sum_{m=1}^{\infty} :\tilde{\alpha}_m^i \tilde{\alpha}^i_{-m}: \qquad \text{level matching}$$

$$\frac{l_s}{\sqrt{2}} p^+ \alpha_n^- = \sum_m \alpha_m^i \alpha_{n-m}^i$$

$$\frac{l_s}{\sqrt{2}} p^+ \tilde{\alpha}_n^- = \sum_m \tilde{\alpha}_m^i \tilde{\alpha}_{n-m}^i$$

• String の Mass Spectrum

M^2

0: $(\alpha^i_{-1}\tilde{\alpha}^j_{-1} + \alpha^j_{-1}\tilde{\alpha}^i_{-1} - \frac{1}{12}\delta^{ij}\alpha^k_{-1}\alpha^k_{-1})|0\rangle$ Graviton $g_{\mu\nu}$

$\oplus\ (\alpha^i_{-1}\tilde{\alpha}^j_{-1} - \alpha^j_{-1}\tilde{\alpha}^i_{-1})|0\rangle$ antisym. tensor $B_{\mu\nu}$

$\oplus\ \alpha^k_{-1}\alpha^k_{-1}|0\rangle$ dilaton ϕ

$-\frac{4}{\ell_s^2}$: $|0\rangle$ tachyon T

• 低エネルギー有効作用

$$S = \frac{1}{2\kappa^2}\int d^{26}x\,\sqrt{-g}\,e^{-2\phi}\left(R + 4\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2\cdot 3!}H_{\mu\nu\rho}H^{\mu\nu\rho}\right)$$

$B_{\mu\nu} \sim U(1)$ ゲージ場 , $H_{\mu\nu\rho} = 3\,\partial_{[\mu}B_{\nu\rho]}$

B場の電荷や磁荷を持つ物体は？

	電磁気	Bosonic
電荷	$e\int A$: electron	$T\int B$: string
磁荷	$m\int \tilde{A}$: magnetic monopole	$T_{21}\int \tilde{B}$: NS21-brane

$F = dA$ $H = dB$

$\tilde{F} = d\hat{A}$ $\hat{H} = d\tilde{B}$

$\hat{F} = \overset{4}{*}F$ $\tilde{H} = \overset{26}{*}H$

Superstring

- Supersymmetry の必要性

Bosonic string にはエネルギー負の状態が存在

これを回避するには何か新しい対称性が必要

$\longrightarrow$ Supersymmetry

- 2 dim. world-sheet を超対称化 $\longrightarrow$ NS-R 形式
- 10 dim. target 空間 〃 $\longrightarrow$ GS 形式

GS 形式で考える.

$$\underbrace{X^\mu(\xi)}_{10\,d\ \text{vector}}\ ,\ \underbrace{S^1(\xi),\ S^2(\xi)}_{10\,d\ \text{Majorana Weyl spinors}}$$

- Superstring の作用

$$\begin{cases} \delta X^\mu = \bar{\epsilon}^A \gamma^\mu S^A \\ \delta S^A = \epsilon^A \end{cases}$$

$\Pi^\mu = dX^\mu - \bar{S}^A \gamma^\mu dS^A$ は超対称不変

$$S = -\frac{T}{2}\int d^2\xi \sqrt{-g}\, g^{\alpha\beta}\partial_\alpha \Pi^\mu \partial_\beta \Pi^\nu \eta_{\mu\nu}$$

$$-T\int (d\bar{S}^1\gamma_\mu S^1 - d\bar{S}^2\gamma_\mu S^2)\Pi^\mu + d\bar{S}^1\gamma_\mu S^1\, d\bar{S}^2\gamma^\mu S^2 \quad \text{WZ 項}$$

WZ 項は $\underline{\kappa\ \text{対称性}}$ を導入するために必要

boson と fermion の自由度を合わせる

• Light-Cone ゲージ

2 dim. Diffeo, Weyl, Conformal, κ 対称性を固定

$$g_{\alpha\beta} = \eta_{\alpha\beta}, \qquad X^+ = x^+ + l_s^2 p^+ t$$

$$\gamma^+ S^1 = \gamma^+ S^2 = 0$$

作用

$$S = \int d\xi^+ d\xi^- \left(T \partial_+ X^i \partial_- X^i + \frac{i}{2\pi} S^1 \partial_+ S^1 + \frac{i}{2\pi} S^2 \partial_- S^2 \right)$$

$$\partial_+\partial_- X^i = 0 \qquad \partial_+ S^1 = 0 \qquad \partial_- S^2 = 0$$

closed $\begin{cases} S^1(t,\sigma+2\pi) = S^1(t,\sigma) \\ S^2(t,\sigma+2\pi) = S^2(t,\sigma) \end{cases}$

量子化

$$\begin{cases} X^i = x^i + l_s^2 p^i t + \dfrac{i l_s}{\sqrt{2}} \displaystyle\sum_{n\neq 0} \frac{1}{n} \left(\alpha_n^i e^{-in(t+\sigma)} + \tilde{\alpha}_n^i e^{-in(t-\sigma)} \right) \\ S^1 = \dfrac{1}{\sqrt{2}} \displaystyle\sum_n \tilde{S}_n e^{-in(t-\sigma)} \qquad \{\tilde{S}_m^a, \tilde{S}_{-n}^b\} = \delta_{m,n} \delta^{a,b} \\ S^2 = \dfrac{1}{\sqrt{2}} \displaystyle\sum_n S_n e^{-in(t+\sigma)} \qquad \{S_m^a, S_n^b\} = \delta_{m,n} \delta^{a,b} \end{cases}$$

拘束条件

$$\begin{cases} M^2 = \dfrac{2}{l_s^2} \displaystyle\sum_{m=1}^{\infty} \left(\alpha_{-m}^i \alpha_m^i + m S_{-m}^a S_m^a + \tilde{\alpha}_{-m}^i \tilde{\alpha}_m^i + m \tilde{S}_{-m}^a \tilde{S}_m^a \right) \quad \text{mass} \\ \displaystyle\sum_{m=1}^{\infty} \left(\alpha_{-m}^i \alpha_m^i + m S_{-m}^a S_m^a \right) = \sum_{m=1}^{\infty} \left(\tilde{\alpha}_{-m}^i \tilde{\alpha}_m^i + m \tilde{S}_{-m}^a \tilde{S}_m^a \right) \quad \text{level match} \end{cases}$$

tachyon なし

$M^2 = 0$ の状態は $S_0^a \tilde{S}_0^b \cdots |0\rangle$

SO(8) の 8_s or 8_c 表現

- SO(8) の triality

$$\left\{\begin{array}{lll} S_0^a \text{ が } 8_v & \longrightarrow & \{S_0^a, S_0^b\} = \delta^{ab} \quad \text{クリフォード代数} \\ & & 16 = 8_s \oplus 8_c \\ S_0^a \text{ が } 8_s & \longrightarrow & 16 = 8_v \oplus 8_c \\ S_0^a \text{ が } 8_c & \longrightarrow & 16 = 8_v \oplus 8_s \end{array}\right.$$

- Type IIA

$$S_0^a \sim 8_c \quad , \quad \tilde{S}_0^a \sim 8_s$$

$$(8_v \oplus 8_s) \otimes (8_v \oplus 8_c)$$

$$= (1 \oplus 28 \oplus 35) \oplus (8_v \oplus 56) \oplus (128 \text{ spinors})$$

$$\begin{array}{ccccc} \phi & B_{\mu\nu} & g_{\mu\nu} & C_\mu & C_{\mu\nu\rho} \\ & \wr & & \wr & \wr \\ & \text{string} & & \text{D0} & \text{D2} \\ & \text{NS5} & & \text{D6} & \text{D4} \end{array}$$

$$\left\{\begin{array}{l} dB^{(2)} = \overset{10}{*} dB^{(6)} \\ dC^{(3)} = \overset{10}{*} dC^{(5)} \\ dC^{(1)} = \overset{10}{*} dC^{(7)} \end{array}\right.$$

- Type IIB

$$S_0^a \sim 8_c \quad , \quad \tilde{S}_0^a \sim 8_c$$

$$(8_v \oplus 8_s) \otimes (8_v \oplus 8_s)$$

$$= (1 \oplus 28 \oplus 35) \oplus (1 \oplus 28 \oplus 35) \oplus (128 \text{ spinors})$$

$$\begin{array}{cccccc} \phi & B_{\mu\nu} & g_{\mu\nu} & C & C_{\mu\nu} & C^+_{\mu\nu\rho\sigma} \\ & \wr & & \wr & \wr & \wr \\ & \text{string} & & \text{D(-1)} & \text{D1} & \text{D3} \\ & \text{NS5} & & \text{D7} & \text{D5} & \end{array}$$

T-duality

- X^1 方向を半径 R の S^1 でコンパクト化

境界条件 : $X^1(t,\sigma+2\pi) = X^1(t,\sigma) + 2\pi R \underbrace{w}_{\text{winding \#}}$

$$X^1(t,\sigma) = x^1 + \underbrace{l_s^2 \underbrace{\frac{m}{R}}_{\text{momentum}} t + Rw\sigma}_{\left(\frac{l_s^2 m}{2R}+\frac{Rw}{2}\right)(t+\sigma)+\left(\frac{l_s^2 m}{2R}-\frac{Rw}{2}\right)(t-\sigma)} + \frac{i l_s}{\sqrt{2}} \sum_{n\neq 0} \frac{1}{n}\left(\alpha_n^1 e^{-in(t+\sigma)} + \tilde{\alpha}_n^1 e^{-in(t-\sigma)}\right)$$

拘束条件は

$$M^2 = M_L^2 + M_R^2 \quad , \quad M_L^2 = M_R^2$$

$$M_R^2 = \frac{1}{2}\left(\frac{m}{R} + \frac{Rw}{l_s^2}\right)^2 + \frac{2}{l_s^2}\sum_{m=1}^{\infty}\left(\alpha_{-m}^i \alpha_m^i + m S_{-m}^a S_m^a\right)$$

$$M_L^2 = \frac{1}{2}\left(\frac{m}{R} - \frac{Rw}{l_s^2}\right)^2 + \frac{2}{l_s^2}\sum_{m=1}^{\infty}\left(\tilde{\alpha}_{-m}^i \tilde{\alpha}_m^i + m \tilde{S}_{-m}^a \tilde{S}_m^a\right)$$

この Mass spectrum は

$$m \longleftrightarrow w$$

$$R \longleftrightarrow \frac{l_s^2}{R}$$

$$\tilde{\alpha}_n^1 \longleftrightarrow -\tilde{\alpha}_n^1 \quad , \quad \tilde{S}_m \longleftrightarrow \gamma\gamma^1 \tilde{S}_m$$

の入れかえで不変　T-duality

上記は次のように書ける

$$T^1 : \left(\begin{array}{l} X^1(t-\sigma) \longrightarrow -X^1(t-\sigma) \\ S(t-\sigma) \longrightarrow \gamma\gamma^1 S(t-\sigma) \end{array} \right.$$

chirality flip

$8_s \longleftrightarrow 8_c$

IIA と IIB は T-duality で結びついている

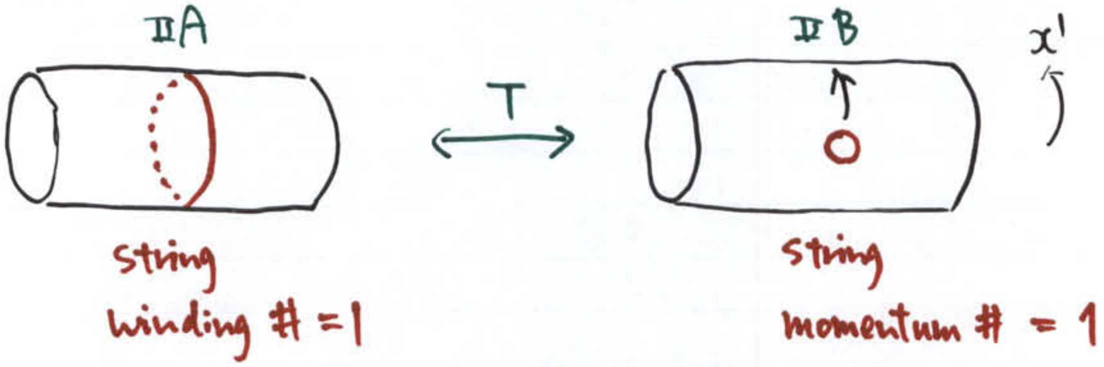
• String や D-branes は T-duality でどうなる？

IIAから始める

$\underline{8_v \otimes \tilde{8}_v}$ $\qquad \tilde{v}^1 \to -\tilde{v}^1$

$g^{\mu\nu} \sim v^\mu \tilde{v}^\nu + v^\nu \tilde{v}^\mu \qquad g^{1i} \to B^{1i}$

$B^{\mu\nu} \sim v^\mu \tilde{v}^\nu - v^\nu \tilde{v}^\mu \qquad B^{1i} \to g^{1i}$



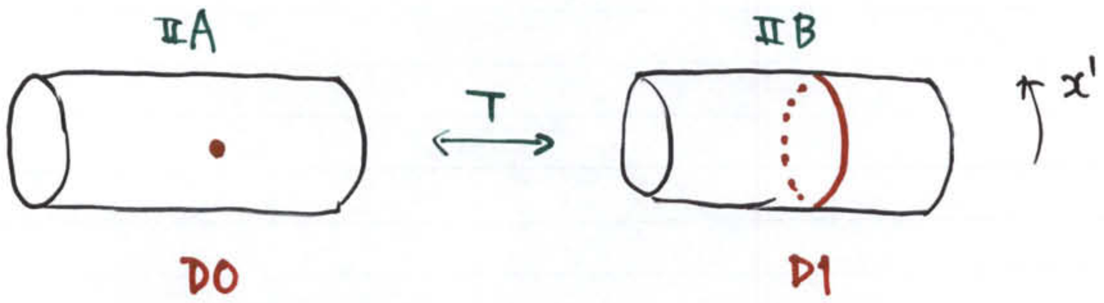
$\underline{8_s \otimes \tilde{8}_c}$ $\qquad \tilde{\theta}_c \to \gamma\gamma^1 \tilde{\theta}_s$

$C^\mu \sim \bar{\theta}_s \gamma^\mu \tilde{\theta}_c \qquad C^1 \to C$

$\qquad C^i \to C^{i1}$

$C^{\mu\nu\rho} \sim \bar{\theta}_s \gamma^{\mu\nu\rho} \tilde{\theta}_c \qquad C^{ij1} \to C^{ij}$

$\qquad C^{ijk} \to C^{ijk1}$



S-duality

- IIB string の低エネルギー有効理論 ～ IIB SUGRA

IIB SUGRA は $SL(2,\mathbb{R})$ の作用で不変

$$
\left\{
\begin{aligned}
&\tau' = \frac{a\tau + b}{c\tau + d}, \qquad \tau = C^{(0)} + \frac{i}{e^{\phi}}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{R}) \\
&\begin{pmatrix} B' \\ C^{(2)\prime} \end{pmatrix} = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \begin{pmatrix} B \\ C^{(2)} \end{pmatrix}
\end{aligned}
\right.
$$

String や D1-brane の charge は量子化されているので
IIB string には $SL(2,\mathbb{Z})$ の対称性があるだろう

特に $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ とすると

$$
\left\{
\begin{aligned}
&\tau' = -\frac{1}{\tau} \qquad C^{(0)} = 0 \text{ のとき } e^{\phi'} = \frac{1}{e^{\phi}} \\
&\begin{pmatrix} B' \\ C^{(2)\prime} \end{pmatrix} = \begin{pmatrix} C^{(2)} \\ -B \end{pmatrix}
\end{aligned}
\right.
$$

Strong-weak duality

- String や D-brane に対しては

$$F1 \xleftrightarrow{S} D1$$

$$NS5 \longleftrightarrow D5$$

$$\circlearrowright\ D3 \quad \longleftarrow \quad \text{4D } \mathcal{N}=4 \text{ SYM の S-duality と一致!}$$

Non Abelian Gauge Symmetry

1. D-branes 上で実現　　Open string
2. 内部空間の幾何で実現　　Closed String

• D-branes 上のゲージ理論 (U(1))

D-brane 上に端を持つ open string

により D-brane は dynamical になる

境界条件 : (N) $X^{\mu\prime}(t,\sigma) = 0 \quad (\mu = 0, 1, \cdots, p)$

$(\sigma = 0, \pi)$ (D) $\dot{X}^{i}(t,\sigma) = 0 \quad (i = p+1, \cdots, 9)$

$$S^1(t,\sigma) = S^2(t,\sigma)$$

Note

$$\left(\begin{aligned} X^{\mu\prime} &= \partial_+ X^\mu(t+\sigma) - \partial_- X^\mu(t-\sigma) = 0 \\ \dot{X}^i &= \partial_+ X^i(t+\sigma) + \partial_- X^i(t-\sigma) = 0 \end{aligned} \right.$$

T-dual $\left(X^i(t-\sigma) \to -X^i(t-\sigma) \right)$ で

N と D は入れかわる

量子化

$$X^\mu = x^\mu + 2 l_s^2 p^\mu t + \frac{i l_s}{\sqrt{2}} \sum_{n \neq 0} \frac{1}{n} \left(\alpha_n^\mu e^{-in(t+\sigma)} + \alpha_n^\mu e^{-in(t-\sigma)} \right)$$

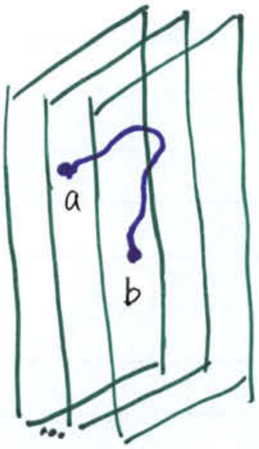
$$X^i = x^i + 2 l_s^2 w^i \sigma + \frac{i l_s}{\sqrt{2}} \sum_{n \neq 0} \frac{1}{n} \left(-\alpha_n^i e^{-in(t+\sigma)} + \alpha_n^i e^{-in(t-\sigma)} \right)$$

$$S^1 = \frac{1}{\sqrt{2}} \sum_n S_n e^{-in(t-\sigma)}$$

$$S^2 = \frac{1}{\sqrt{2}} \sum_n S_n e^{-in(t+\sigma)}$$

$$\underbrace{8_v}_{A_\mu,\, X^i} \oplus 8_s \qquad U(1)$$

- D-branes 上のゲージ理論 (U(N))



1 … N

D-branes が N 枚あったとすると
Open string の端点がどの D-brane
上にあるかを区別する自由度が
生じる → Chan-Paton 因子
N×N 通り

$$A_\mu{}^a{}_b\ ,\quad X^{i\,a}{}_b\ ,\quad \Theta^a{}_b \longrightarrow U(N)\ \mathrm{SYM}$$

- Type I

IIB + D9×16 の系で left と right の入れかえ Ω
に対して対称なものだけを拾ってくる

→ Type I

closed string

$$(8_v \oplus 8_s) \underset{\mathrm{sym}}{\otimes} (8_v \oplus 8_s) = (\underset{\phi}{1} \oplus \underset{\underset{D1,\,D5}{\wr}}{\underset{C_{\mu\nu}}{28}} \oplus \underset{g_{\mu\nu}}{35}) \oplus (64\ \mathrm{fermion})$$

open string on D9×16 + O9$^-$

$$(8_v \oplus 8_s)_{ab} \qquad SO(32)\ \mathrm{SYM}$$

$a,b = 1 \sim 32$

- Heterotic string $SO(32)$ or $E_8 \times E_8$

Non abelian gauge symmetry を持つ
これは内部空間の幾何構造から決まる

right ~ Superstring $X^\mu(t+\sigma)$, $S(t+\sigma)$

left ~ Bosonic string $X^\mu(t-\sigma)$, $X^I(t-\sigma)$

$\mu = 0 \sim 9$ $\quad I = 1 \sim 16$

$I = 1 \sim 16$ でラベルされる空間をトーラス T^{16} でコンパクト化

$$T^{16} = \mathbb{R}^{16} / \Gamma$$

Γ が even, self-dual lattice のとき矛盾が生じない

$$\Gamma \sim \underbrace{\Gamma_8 \times \Gamma_8}_{E_8 \text{ root lattice}} \text{ or } \underbrace{\Gamma_{16}}_{SO(32) \text{ root lattice}}$$

- 簡単な例で感じをつかもう

Bosonic string を S^1 コンパクト化した場合

$$M^2 = M_L^2 + M_R^2 \quad , \quad M_L^2 = M_R^2$$

$$M_R^2 = \frac{2}{l_s^2} \sum_{m=1}^{\infty} \alpha_{-m}^i \alpha_m^i + \frac{1}{2}\left(\frac{m}{R} + \frac{Rw}{l_s^2}\right)^2 - \frac{2}{l_s^2}$$

$$M_L^2 = \frac{2}{l_s^2} \sum_{m=1}^{\infty} \tilde{\alpha}_{-m}^i \tilde{\alpha}_m^i + \frac{1}{2}\left(\frac{m}{R} - \frac{Rw}{l_s^2}\right)^2 - \frac{2}{l_s^2}$$

$M^2 = 0$ となるのは

- $(\alpha_{-1}^i \tilde{\alpha}_{-1}^z \pm \alpha_{-1}^z \tilde{\alpha}_{-1}^i) |\overset{m}{0}, \overset{w}{0}\rangle$ $\quad B^{iz}$, g^{iz} $\qquad U(1) \times U(1)$

$\qquad \downarrow R = l_s$

- $R = l_s$ $\alpha_{-1}^i |1,-1\rangle$, $\alpha_{-1}^i |-1,1\rangle$, $\tilde{\alpha}_{-1}^i |1,1\rangle$, $\tilde{\alpha}_{-1}^i |-1,-1\rangle$ $\qquad SU(2) \times SU(2)$

- Hetero の mass spectrum

  $M^2=0$ から (∵ level matching)

$$(8_v \oplus 8_s) \otimes 8_v = (\underset{\phi}{1} \oplus \underset{B_{\mu\nu}}{28} \oplus \underset{g_{\mu\nu}}{35}) \oplus (64 \text{ spinors})$$

$B_{\mu\nu}$ ⊂ F1, NS5

$$(8_v \oplus 8_s) \left| \begin{matrix} E_8 \times E_8 \\ SO(32) \end{matrix} \right\rangle \qquad \text{SYM}$$

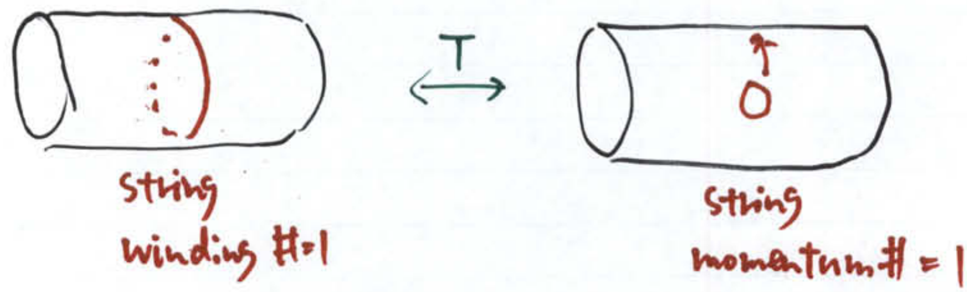
- Hetero SO(32) $\xleftrightarrow{T}$ Hetero $E_8 \times E_8$

  ゲージ群を合わせる

  S^1 でコンパクト化して その方向の bosonic string の

  left を反転してもスペクトラムは不変

$$X^9(t-\sigma) \to -X^9(t-\sigma)$$



- Hetero SO(32) $\xleftrightarrow{S}$ Type I SO(32)

F1	D1
NS5	D5

- Duality web

$$
\begin{tikzcd}
11\mathrm{D} & & & \boxed{\mathrm{M}} \arrow[dll, "S^1"'] \arrow[drr, "S^1/\mathbb{Z}_2"] & & \\
10\mathrm{D} & \boxed{\mathrm{IIA}} \arrow[r, leftrightarrow, "T"] & \boxed{\mathrm{IIB}} \arrow[loop above, "S"] \arrow[r, leftrightarrow, "\Omega"] & \boxed{\mathrm{Type\ I}} \arrow[r, leftrightarrow, "S"] & \boxed{\mathrm{Het}\ SO(32)} \arrow[r, leftrightarrow, "T"] & \boxed{\mathrm{Het}\ E_8\times E_8}
\end{tikzcd}
$$

- 双対性によってString理論は結びつくと考えるのは自然

- D-braneの導入が不可欠

- 11Dの理論もある. M理論